How the SP Theory of Intelligence and its realisation in the SP Computer Model may be applied with advantage in science

J Gerard Wolff*

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Abstract

This paper describes how information compression via the SP Theory of Intelligence (SPTI) and its realisation in the SP Computer Model (SPCM) may be applied, with advantage, in science In the main sections: 1) describes briefly some of the potential benefits of the SPTI in science; 2) describes how the SPTI may be integrated with mathematics, and potential benefits in science from that integration; 3) discusses how the SPTI provides principles which may help to overcome known problems with the concept of 'infinity' in physics; 4) discusses some misconceptions about the concept of 'superposition' in quantum mechanics (QM), and shows with an example from the SPCM how the QM concept of superposition shares features with the concept of 'syntactic class' in theoretical linguistics. Similar things may be said about the concept of 'data type' in mathematics and computing. As a possible resolution of the counter-intuitive idea that quantum entities do not exist until they are measured or detected, this section introduces a tentative 'tsunami' model of wave-particle duality. This section also discusses potential implications of what has been described for concepts in 'quantum computing'; 5) With an example from the SPCM, this section describes how the QM concepts of 'nonlocality' and 'entanglement' may be modelled via the powerful concept of SP-multiple-alignment in the SPTI; 6) In the light of principles in the SPTI and its foundations, this section discusses what appear to be fatal weaknesses in the 'Many Worlds' interpretation of QM, and associated concepts.

^{*}Dr Gerry Wolff BA (Cantab) PhD (Wales) CEng MIEEE MBCS; ORCID ID: 0000-0002-4624-8904; CognitionResearch.org, UK; jgw@cognitionresearch.org; +44 (0) 1248 712962; +44 (0) 7746 290775; *Twitter*: @gerrywolff65; *Web*: www.cognitionresearch.org/sp.htm.

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1 Introduction

This paper describes how the SP Theory of Intelligence (SPTI), and its realisation in the SP Computer Model (SPCM), may be applied with advantage in science.

The SPTI is the product of a lengthy programme of research, seeking to simplify and integrate observations and concepts across Artificial Intelligence (AI), mainstream computing, mathematics, and Human Learning, Perception, and Cognition (HLPC).

A central idea in the SPTI is information compression (IC) as a unifying principle across AI, mainstream computing, mathematics, and human learning, perception, and cognition (HLPC). That principle derives from evidence for the importance of IC in HLPC [69].

A working hypothesis in this research is that IC may in principle always be understood in terms of a search for patterns that match each other and the merging or 'unification' patterns that are the same (Appendix C.2). The expression 'Information Compression via the Matching and Unification of Patterns' is abbreviated as 'ICMUP'.

The SPTI, with the SPCM, is described in outline in Appendix A with pointers to where fuller information may be found. Readers who are not already familiar with the SP concepts are urged to read Appendix A, and perhaps some of the other sources, before reading anything else.

1.1 Presentation

The main sections of the paper are these:

- Potential benefits of the SPTI for science, in brief. Section 2 outlines some of the potential benefits of the SPTI for science.
- *Mathematics and the SPTI*. Section 3 outlines evidence presented in [70] showing that much of mathematics, perhaps all of it, may be understood as IC. On the strength of that evidence, it is argued that there is potential for the integration of mathematics with the SPTI, and there are potential benefits for science from that integration.
- SPTI as an aid to non-mathematical thinking. Section 4 describes how the SPTI may be an aid to the kind of non-mathematical thinking that appears often to be the key to success in the work of leading scientists.

- Overcoming problems with infinity. Section 5 describes how the SPTI may help overcome some of the known problems with the concept of 'infinity' in science.
- Issues with the quantum mechanics concept of superposition. Section 6 discusses what appear to be some misconceptions about the concept of 'superposition' in quantum mechanics (QM), and shows with an example from the SPCM how the QM concept of superposition shares features with the concept of 'syntactic class' in theoretical linguistics, and the concept of 'data type' in mathematics and computing.

As a possible resolution of the counter-intuitive idea that quantum entities do not exist until they are measured or detected, this section introduces a tentative 'tsunami' model of wave-particle duality. This section also discusses potential implications for concepts in 'quantum computing';

- Non-locality, entanglement, and SP-multiple-alignment. With an example from the SPCM, Section 7 describes how the QM concepts of 'nonlocality' and 'entanglement' may be modelled via the powerful concept of SP-multiple-alignment in the SPTI.
- Weaknesses in the 'Many Worlds' interpretation of QM. In the light of principles in the SPTI and its foundations, Section 8 discusses what appear to be fatal weaknesses in the 'Many Worlds' interpretation of QM.

2 Some of the potential benefits of the SPTI for science, in brief

Some of the potential benefits of the SPTI for science are summarised here, drawing on and adapting what is described in [70, Section 9.2].

2.1 AI-related strengths and potential

The strengths and potential of the SPTI in aspects of artificial intelligence (Appendix A.5) are likely to prove useful in many areas of science.

2.2 Potential for the SPTI to achieve quantitative evaluation of scientific theories

In scientific research as it has been up to now, the evaluation of rival scientific theories has been done via more-or-less informal debate, and it seems likely that this will be true for some time to come. Because IC is at the heart of the SPTI, there is potential for the SPTI to achieve quantitative evaluation of scientific theories in terms of IC.

2.3 Potential in unsupervised learning

There is also potential in unsupervised learning in the SPTI (Appendix A.4) for the automatic or semi-automatic development of scientific theories from appropriate data.

2.4 Potential in the representation of knowledge

There is potential in the SPTI to be developed as a *universal framework for the* representation and processing of diverse kinds of knowledge(UFK) [63, Section III].

There is potential in this idea to minimise the diversity of formats and formalisms which are commonly needed to represent different kinds of knowledge, and to minimise the need for each format or formalism to be processed by its own specialised software or hardware.

2.5 Potential for the integration of mathematics, logic, and computing

There is also potential in the SPTI to facilitate the integration of mathematics, logic, and computing [70, Section 7].

2.6 Potential for alternative perspectives in statistics

Because of the close relation between IC and concepts of probability, there is potential for some alternative perspectives in statistics, some of which are described in Appendix C.4.

3 Potential benefits in science from the integration of mathematics with the SPTI

Here are the main reasons for supposing that mathematics may be integrated with the SPTI, as it is now and as it may be developed in the future:

• There is evidence that much of HLPC may be understood as ICMUP (Appendix C.3).

- The SPTI draws much of its inspiration from the importance, just mentioned, of ICMUP in HLPC, and all its processing is achieved via ICMUP (Appendix A.1).
- Since mathematics is the product of human minds, and has been developed as an aid to human thinking, it should not be surprising to find that much of mathematics, perhaps all of it, may be understood as ICMUP [70].

Here are two examples of how mathematics may be understood as ICMUP:

- Run-length coding, which is a widely-used technique for ICMUP (Appendix C.2.4), means that any sequence of two or more copies of a pattern may be reduced to one, with some indication that the pattern repeats. Since a multiplication like 5×4 may be understood as 5 repetitions of "add 4 to x" (starting with "set x to be 0"), it may also be seen as an instance of run-length coding.
- Chunking-with-codes, which is another widely-used technique for ICMUP (Appendix C.2.2), means using a relatively-short 'code' or name for a relatively-large 'chunk' of information. In an expression like sqrt(16), 'sqrt' is a relatively-short code for the relatively large program (the 'chunk') that calculates square roots. It makes no difference that square roots are commonly written as $\sqrt{16}$: the $\sqrt{}$ symbol means the same as sqrt.

In these examples, 16 is simply data for the calculation and not part of the concept of chunking-with-codes.

• Since ICMUP is fundamental in the SPTI, and appears to be fundamental in mathematics [70], there is clear potential for the integration of the SPTI with mathematics via the shared currency of ICMUP.

In general, the potential benefits of such an integration of mathematics and the SPTI is that the integrated system would have more strengths than either of them alone:

- The SPTI may gain from the enormous knowledge in mathematics, and the many techniques not currently part of the SPTI, such as differential and integral calculus, matrix multiplication, and more.
- Mathematics may gain strengths in AI that are not normally considered to be part of mathematics, and more generally from the power of the 'SP-multiple-alignment' concept (Appendix A.3), something that is currently missing from mathematics.

Since the strengths of the SPTI are less well known than the strengths of mathematics, this paper concentrates on the potential benefits of the SPTI for science.

4 The SPTI as an aid to non-mathematical thinking by leading scientists and others

There is quite a lot of evidence that, although mainstream mathematics is often useful for the representation and processing of scientific laws and theories, it may not be so good for the thinking behind those laws and theories. It seems that leading scientists, and probably others, often think in a medium that may be roughly characterised as a 'visual' or 'non-mathematical' medium which is not the same as branches of mathematics such as geometry or topology.

Evidence in support of these points, including some from [70, Section 9.2.2], is summarised in subsections that follow.

4.1 How the SPTI may model the kinds of 'visual' thinking exhibited by leading scientists and others

This section outlines evidence for the potential of the SPTI to be an aid to the kind of 'visual' thinking which seems to be a special strength of some leading scientists, and others—and to do it via relatively simple applications of ICMUP, in accordance with evidence for the importance of ICMUP in HLPC [69].

Here, in brief, are some possibilities:

- Since, as anticipated (Appendix A), the SPTI will be developed to process SP-patterns in two dimensions, it should, with that development, be relatively straightforward to represent pictures and diagrams in two dimensions, and to process them via ICMUP.
- Again, capabilities with 2D SP-patterns will allow the system to discover and process structures in three dimensions. How this may be done is described in [69, Sections 6.1 and 6.2].

In brief, 2D pictures that overlap each other may be taken of a 3D object from several different angles and then neighboring pictures that overlap each other may be stitched together via ICMUP in the same way that panoramas may be built up from two or more pictures of a scene that are partially overlapping, one with the next. There are now several commercial companies that can create 3D digital objects in this way. • With 2D SP-patterns, there is also potential for the representation of processes that occur in parallel, and their processing via ICMUP [62, Sections V-G, V-H, and V-I, and Appendix C].

Of course, we do not yet have precise ideas about how leading scientists and others think with pictures and 3D structures. But what has been outlined is distinctly different from representations via the mathematical disciplines of geometry and topology. As relatively direct applications of ICMUP, they seem to be a better fit with evidence for the importance of ICMUP in HLPC (Appendix C.3).

4.2 "I've never been good at mathematics"

Many people say, sometimes with pride, that they have never been good at mathematics and that includes people who have been successful in careers that require high intelligence, with the implication that they are using non-mathematical structures and processes in their thinking.

4.3 "Every equation will halve your sales"

As is well known, Stephen Hawking was told by the publisher of the first edition of his book, *A Brief History of Time* [24], that "... every equation will halve your sales." [55, location 3227], with the same implication as before.

4.4 The mind's eye

Carlo Rovelli writes that "Einstein had a unique capacity to imagine how the world might be constructed, to 'see' it in his mind." [43, location 1025].

In that connection:

• Walter Isaacson writes that Einstein "repeatedly said that his path toward the theory of relativity began with his thought experiment at age 16 about what it would be like to ride at the speed of light alongside a light beam. This produced a 'paradox,' he said, and it troubled him for the next ten years." [27, location 2207].

The puzzling question was whether, when he was travelling at the speed of light, he would perceive the speed of light as zero or its normal speed. In the end, he decided, probably because it was necessary to achieve a parsimonious theory of relativity, that the speed of light (in a vacuum) would be the same, however fast one was travelling.

• Here is another example, written by Albert Einstein himself, of non-mathematical simplicity in his thinking:

"It is not clear what is to be understood ... by 'position' and 'space'. I stand at the window of a railway carriage which is travelling uniformly, and drop a stone on the embankment, without throwing it. Then, disregarding the influence of the air resistance, I see the stone descend in a straight line. A pedestrian who observes the misdeed from the footpath notices that the stone falls to earth in a parabolic curve. I now ask: Do the 'positions' traversed by the stone lie 'in reality' on a straight line or on a parabola? Moreover, what is meant here by motion 'in space'?" [18, p. 8].

In case these quotes give the impression that Einstein was not good at mathematics: "In 1935, a rabbi in Princeton showed [Einstein] a clipping of [a newspaper column] with the headline 'Greatest Living Mathematician Failed in Mathematics.' Einstein laughed. 'I never failed in mathematics,' he replied, correctly. 'Before I was fifteen I had mastered differential and integral calculus.'" [27, location 488].

4.5 Scientists don't always use mathematics

As the title of this subsection says, scientists don't always use mathematics:

- The distinguished geneticist, Professor Steve Jones FRS, has said that when he comes to equations in scientific papers, he "hums" them.
- It appears that Michael Faraday developed his ideas about electricity and magnetism with little or no knowledge of mathematics: "Without knowing mathematics, [Faraday] writes one of the best books of physics ever written, virtually devoid of equations. He sees physics with his mind's eye, and with his mind's eye creates worlds." [43, location 623].
- Charles Darwin's and Alfred Russel Wallace's theory of evolution by natural selection was developed and published as words and pictures [13], and is still normally presented in that form (but see Gregory Chaitin's proposal for a mathematical theory of evolution by natural selection [10]).

4.6 Feynman diagrams

The famous diagrams invented by Nobel-prize-winning physicist Richard Feynman, outside the realms of conventional mathematics, provided and still provide a means of representing concepts in particle physics which greatly simplify the associated calculations. Martinus Veltman writes: "In 1948 quantum mechanics entered a new phase. Increasingly precise experimental results required new calculation methods, as the existing methods were hopelessly inadequate to deal with the complications of the theory. Richard Feynman came up with a new method that led to enormous simplifications. The method relied heavily on little drawings, now called Feynman diagrams. For a given situation one would draw a few of these diagrams, and then there were simple rules that provided the calculational answers in connection with them. As these diagrams are moreover very appealing intuitively they have become the universal tools of particle physics." [52, p. 244].

4.7 Commonsense reasoning and commonsense knowledge

It appears that the kinds of 'visual' or 'non-mathematical' thinking described above, are essentially what researchers are trying to understand in the important field of AI and cognitive science known as 'commonsense reasoning and commonsense knowledge' (CSRK) [15].

There are features of our everyday thinking and everyday actions which have proved to be remarkably difficult to describe: in mathematics; in classical logic; in non-classical logics such as fuzzy logic, intuitionistic logic, relevance logic, and more; and in many other approaches to AI.

An example of the kind of feature of everyday life which has proved so difficult to describe with formal systems is the seemingly simple task of cracking an egg into a bowl. This is one of the examples contributed by Ernest Davis to the 'Commonsense Reasoning Problem Page' (bit.ly/2qjdMBj), on the website about 'Commonsense Reasoning' (bit.ly/2CPMWbq, retrieved 2022-07-18). Davis describes the problem like this:

"Characterize the following: A cook is cracking a raw egg against a glass bowl. Properly performed, the impact of the egg against the edge of the bowl will crack the eggshell in half. Holding the egg over the bowl, the cook will then separate the two halves of the shell with his fingers, enlarging the crack, and the contents of the egg will fall gently into the bowl. The end result is that the entire contents of the egg will be in the bowl, with the yolk unbroken, and that the two halves of the shell are held in the cook's fingers." (retrieved, 2022-07-18).

Despite the apparent simplicity of cracking an egg into a bowl, three different attempts at formalising the process [30, 33, 46] show how complicated it can be to represent the process in the style of mathematics or logic.

Why should the SPTI do any better? In its current state of development, it cannot provide a satisfactory description of the egg-cracking problem and is unlikely to do much better with many other kinds of CSRK. But the clear potential of the SPTI to help solve 20 significant problems in AI (Appendix A.7) provides a reason for believing that the SPTI provides a firmer foundation for the development of AGI, including CSRK, than any alternative. It seems likely that the potential of the SPTI to represent 3D structures (Appendix A.4.4) will be important in any good solution.

5 Infinity in mathematics and science

A prominent feature of mainstream mathematics and other formal systems (including the SPTI), and the subject of highly-original research by Georg Cantor (see, for example, [14]), is the ability to describe things that are infinitely big or infinitely small.

This is a feature of the number system and can be done quite simply with a recursive function like the computer function in C or C++ shown in Figure 1. This function has the potential to print an infinitely long sequence of 1s, provided: it does not exhaust the available memory in the computer that is to process the function; and it does not run out of paper; and the computer does not crash; and so on.

```
void infinity()
{
    printf{"%d ", '1'};
    return infinity();
}
```

Figure 1: A simple recursive function in the C or C++ computer language with the potential to print an infinitely long sequence of 1s.

Although it can be fascinating to consider how infinities may come in different sizes, and paradoxes that arise from the concept of infinity, the notion of infinity can be problematic for science, leading to predictions that things in the world might be infinitely big or infinitely small—predictions that may be queried.

Here we first quote what some scientists say about such problems, and then we show how the SPTI may provide some answers.

5.1 What some scientists say about the concept of infinity in science

Max Tegmark writes:

"The assumption that something truly infinite exists in nature underlies every physics course I've ever taught at MIT—and, indeed, all of modern physics. But it's an untested assumption, which begs the question: is it actually true?" [50, location 804].

He goes on to say:

"There are in fact two separate assumptions: 'infinitely big' and 'infinitely small.' By infinitely big, I mean that space can have infinite volume, that time can continue forever, and that there can be infinitely many physical objects. By infinitely small, I mean the continuum—the idea that even a liter of space contains an infinite number of points, that space can be stretched out indefinitely without anything bad happening, and that there are quantities in nature that can vary continuously. The two assumptions are closely related, because inflation, the most popular explanation of our Big Bang, can create an infinite volume by stretching continuous space indefinitely.

"The theory of inflation has been spectacularly successful and is a leading contender for a Nobel Prize. It explains how a subatomic speck of matter transformed into a massive Big Bang, creating a huge, flat, uniform universe, with tiny density fluctuations that eventually grew into today's galaxies and cosmic large-scale structure—all in beautiful agreement with precision measurements from experiments such as the *Planck* and the BI-CEP2 experiments. But by predicting that space isn't just big but truly infinite, inflation has also brought about the so-called measure problem, which I view as the greatest crisis facing modern physics. Physics is all about predicting the future from the past, but inflation seems to sabotage this. When we try to predict the probability that something particular will happen, inflation always gives the same useless answer: infinity divided by infinity. The problem is that whatever experiment you make, inflation predicts there will be infinitely many copies of you, far away in our infinite space, obtaining each physically possible outcome; and despite years of teeth-grinding in the cosmology community, no consensus has emerged on how to extract sensible answers from these infinities. So, strictly speaking, we physicists can no longer predict anything at all!

"This means that today's best theories need a major shakeup by retiring an incorrect assumption. Which one? Here's my prime suspect: ∞ .

"A rubber band can't be stretched indefinitely, because although it seems smooth and continuous, that's merely a convenient approximation. It's really made of atoms, and if you stretch it too far, it snaps. If we similarly retire the idea that space itself is an infinitely stretchy continuum, then a big snap of sorts stops inflation from producing an infinitely big space and the measure problem goes away. Without the infinitely small, inflation can't make the infinitely big, so you get rid of both infinities in one fell swoop—together with many other problems plaguing modern physics, such as infinitely dense black-hole singularities and infinities popping up when we try to quantize gravity." [50, Locations 804–827].

Tegmark acknowledges that "infinity is an extremely convenient approximation for which we haven't discovered convenient alternatives." [50, location 830], but warns that "Despite their seductive allure, we have no direct observational evidence for either the infinitely big or the infinitely small." [50, location 837]. And Jim Al Khalili writes:

And Jim Al-Khalili writes:

"An obvious question, with history as our guide, is whether the electron and quarks are indeed fundamental, or made of yet smaller pieces like Russian dolls. The honest answer is: we don't know! All we can say is that with the best experiments we are able to do today, there is no hint of deeper structure." [1, Locations 2478–2484].

In a similar vein, writing about the elimination of the concept of infinity in the theory of quantum gravity, Rovelli says:

"The infinitely small no longer exists. The infinities which plague conventional quantum field theory, predicated on the notion of a continuous space, now vanish, because they were generated precisely by the assumption, physically incorrect, of the continuity of space. The singularities which render Einstein's equations absurd when the gravitational field becomes too strong also disappear: they are only the result of neglecting the quantization of the field." [43, location 2130].

Thus it is possible to develop a scientific theory without the use of infinities. But merely removing infinities from a theory by fiat does not overcome what appears to be a major weakness in mathematics *as a vehicle for scientific knowledge*, but not necessarily in other areas of application.

Since the focus of this paper is on science, the next subsection (Section 5.2) shows how the SPTI, with the SPTI's basis in IC, can provide principled answers to the problem of infinity in science, preventing inferences that run too far ahead of their empirical underpinnings.

5.2 How the SPTI may help to put a brake on the use of infinity in science

As we have seen at the beginning of Section 5, infinity may be expressed in conventional computing via recursive structures like the function shown in Figure 1. The subsections that follow describe, with examples from the SPCM, two ways in which the SPTI may capture the same kind of recursion. Section 5.2.1 describes how the SPTI may achieve recursive processing without end, like the function shown in Figure 1. Then Sections 5.2.2 and 5.2.3 describe, with examples from the SPCM, how that kind of unrestricted recursive processing is likely to be misleading in modelling entities or phenomena in the 'world', meaning anything in the universe as we know it.

5.2.1 Case 1: How a potentially infinite sequence of 1s may be created recursively by the SPCM

In the example described here, there is one New SP-pattern, '(P Q R)', and two Old SP-patterns: '(X a P Q R #X)' and '(X b X #X 1 #X)', shown in Figure 2.

New SP-pattern P Q R

Old SP-patterns

X a P Q R #X X b X #X 1 #X

Figure 2: New and Old SP-patterns as discussed in the text.

Figure 3 shows two of many similar SP-multiple-alignments (SPMAs, Appendix A.3) created recursively by the SPCM with the SP-patterns in Figure 2. Each of those many similar SPMAs has zero or more 1s on the right-hand side. Here are some points that may help to clarify the example:

- Recursion is achieved because the SP-symbols 'X' and '#X' at the beginning and end of both of the Old SP-patterns ('X a P Q R #X' and 'X b X #X 1 #X') match the same two symbols in the body of the SP-pattern 'X b X #X 1 #X'.
- Although there is only one instance of the SP-pattern '(X b X #X 1 #X)' in the repository of Old SP-patterns, it can appear one, two or more times in

any SPMA. That any one SP-pattern may appear two or more times in any SPMA is an important feature of how SPMAs may be formed, as described in [58, Section 3.4.6].

- In the SPTI there is a distinction between 'ID' SP-symbols, and 'C' SP-symbols:
 - The 'ID', SP-symbols, which are the SP-symbols 'X', '#X', 'a' and 'b' in Figure 3, serve to identify or classify the SP-patterns in which they appear. The ID SP-symbols play a supporting role and may be ignored by anyone whose main interest is in the results of a computation and not how it is done. ID SP-symbols are like such elements of the C or C++ programming languages as 'void', 'printf', and ';'.
 - The 'C', SP-symbols, which are the SP-symbols 'P', 'Q', 'R', and '1', in Figure 3, serve to describe the 'contents' which is being processed by the system. In the interpretation of the examples in Figure 3, it is mainly those four SP-symbols that are of interest.
- For this example to work as described below, a small adaptation is needed in its data. Normally, the ID SP-symbols 'a' and 'b' would be assigned a non-zero positive information value in bits which would become the information 'cost' of the code derived from any SPMA. But in this example, that information value is set to zero so that any and all SPMAs produced by the SPCM yield an encoding cost which is zero.



Figure 3: Two of many similar SPMAs that may be created by the SPCM as described in the text. They show how the SPCM may potentially create an infinite sequence of 1s.

In principle, this kind of recursive processing can proceed without limits, but with the kinds of practical limitations mentioned earlier: exhaustion of the computer's memory, crashing of the computer, and so on. However, as we shall see in Sections 5.2.2 and 5.2.3, there are constraints that are likely to apply in any example that aims to model one or more aspects of the world.

In this example, the SP-symbols 'P', 'Q', and 'R', may be seen to represent established knowledge, meaning things that have been directly observed. The SPsymbol '1', within the SP-pattern '(X b X #X 1 #X)', may be seen to represent an inference that may be made recursively from the established knowledge to create such sequences of SP-symbols as 'P Q R 1 1', 'P Q R 1 1 1 1 1', and so on.

5.2.2 Case 2: The effect of more complexity in stored knowledge

This subsection and the one that follows describe reasons in principle (not merely practical limitations) why the kind of recursive processing without limits that is illustrated by the SPMAs (a) and (b) in Figure 3, may be unrealistic in modelling aspects of the world.

The argument in this subsection is that the world is rarely as precise as mathematics may suggest. This is accepted with things like Boyle's Law (PV = k)where P is the pressure of a gas in a container, V is its volume, k is a constant, and the temperature is constant—which is seen to approximate what is happening with the multitude of molecules in the gas.

With Boyle's law, and laws of that kind, there is a need to bear in mind the possibility that, beneath the apparent precision of the mathematics that describes such things as quarks¹, there may be something more complex and messy.

As an example with the SPTI, our Case 1 example (Section 5.2.1) may be modified to show how a little more complexity in the repository of Old SP-patterns can disturb the purity of its recursive production of a potentially infinite sequence of 1s.

Thus, if the Old SP-pattern '(X c X #X 2 #X)' is added to the Old SP-patterns in Figure 2), and if the SPCM is run with the same New SP-pattern as before ('(P Q R)'), the result is SPMAs exhibiting haphazard sequences of 1s and 2s such as '2 2 1 2 1', '2 2 2 1 2', '1 1 2 2 2', and the like.

In short, a small amount of additional information in the repository of Old SP-patterns may easily add complexity to the sequence of '1s' in Case 1 (Section 5.2.1). This added complexity may disturb the neat inferences that things may be infinitely big or infinitely small, as can happen with over-simplistic applications of mathematics.

5.2.3 Case 3: The effect of taking full account of the compression of information

To create an infinite sequence of SPMAs like those shown in Figure 3, it is necessary for the ID SP-symbol 'a' to have an information cost in bits, C_a , of zero, and, likewise, with the ID SP-symbol 'b' (Section 5.2.1).

If we run the model correctly, so that the information costs of the ID SPsymbols, 'a' and 'b', are greater than zero, the picture is different. The information cost of each SPMA is calculated as described in [60, Section 4.1] and in [58, Section 3.5]. For each SPMA like those shown in Figure 3, the information cost, C_spma is $C_a + (C_b * n)$, where n is the number of 'b's in the SPMA.

Now, for SPMAs like those shown in Figure 3, we can see:

• That C_spma values for SPMAs with a large number of 'b's and '1's will be larger than SPMAs with a small number of 'b's and '1's in any given SPMA.

¹So much so that Tegmark has argued in his book *Our Mathematical Universe* that "our physical world is a giant mathematical object" [51, p. 246].

- The information value of each SPMA is the number of New SP-symbols in the SPMA, which will be 'P Q R' in all cases.
- Hence the IC that is achieved will get progressively worse from SPMAs with a small number of 'b's and '1's compared with SPMAs with a large number of 'b's and '1's.

Depending on the sizes of 'a', 'b', and 'P Q R', it can easily happen that values for IC are negative.

If the number of '1's in an SPMA is seen as an analogy for the size of some physical entity, and if negative values for IC are taken to mean that we have come to the end of meaningful inferences, it is clear that values above zero for the sizes of 'a' and 'b' in our example, can easily put a brake on how far a recursive sequence of inferences may be taken.

In a similar way, it may be argued that there can be a brake on inferences that something might be infinitely small.

5.3 Comment

The evidence and arguments presented in Sections 5.2.2 and 5.2.3, show with examples how the SPCM, within the SPTI, would naturally tend to inhibit inferences that anything in nature was infinitely big or infinitely small.

But, in keeping with Al-Khalili's previously-quoted remarks (Section 5.1), failure to demonstrate infinity in nature says nothing about whether or not it exists. Frustrating as that may be, we should be content to accept an open verdict about whether or not there is infinity in nature, pending further evidence and arguments.

6 Superposition in quantum mechanics

This section argues that the meaning of the word 'superposition' in QM is different from the meaning of that word in wave mechanics, and is essentially the same as the meanings of the expressions 'syntactic class' in theoretical linguistics, and 'data type' in mathematics and computing.

In what follows, some apparent misconceptions about the concept of 'superposition' in QM are described, with suggested clarifications. The SPTI is relevant in so far as it provides a means of expressing those clarifications.

Section 6.4 describes some possible implications of what has been said in this section for concepts associated with 'quantum computing'.

In an appendix, it is suggested that a marine tsunami may provide a useful analogy for understanding aspects of wave-particle duality.

6.1 Some apparent misconceptions about 'superposition' and related ideas in quantum mechanics, and how they may be resolved

The concept of superposition of waves is described by Al-Khalili like this:

"The idea of superposition is not unique to quantum mechanics but is a general property of all waves. Imagine watching someone dive into an empty swimming pool. You will see the ripples travel outwards along the surface of the water as simple undulations all the way to the other end of the pool. This is in stark contrast to the state of the water when the pool is full of people swimming and splashing about. The turbulent shape of its surface is now due to the combined effect of many disturbances and is achieved by adding them all together. This process of adding different waves together is known as superposition." [1, location 1025].

Thus the concept of superposition of physical waves is clear. But *it appears* that, in QM, the word 'superposition' means something different from what it does in wave mechanics. As we shall see, it is applied to the concept of a 'wavefunction', and a wavefunction is not a physical entity, and thus not a physical wave!

Related points are expanded in Sections 6.1.1 to 6.1.4 that follow.

After that, Sections 6.2.1 and 6.2.2 argue that the concept of superposition is not unique to QM, and is much less puzzling and mysterious than is commonly supposed, because it is essentially the same as: 1) the concept of 'syntactic class' in theoretical linguistics; and 2) the concept of 'data type' in mathematics and computing.

6.1.1 Source of confusion: the assumption that a 'wavefunction' is a physical entity like a physical wave

The concept of a 'wavefunction' is described by Ball like this:

"The French physicist Roland Omnès put it nicely when he called the wavefunction "the fuel of a machine that manufactures probabilities" [37, p. 155]. In general, the chance of measuring any particular value of an observable property of a quantum system in an experiment can be calculated by a particular mathematical manipulation of its wavefunction. The wavefunction encodes this information, and quantum maths lets you extract it. There's a particular operation you conduct on the wavefunction to find a particle's momentum (mass × velocity), another

operation to find its energy, and so on. In each case, what you get from this operation is *not* exactly the momentum, or energy, or whatever, that you'd measure in an experiment; it's the *average* value you'd expect to get from many such measurements. ([4, Locations 448–458], emphasis in the original).

However, a source of confusion in QM is when people believe that a 'wavefunction' describes a physical entity. In this connection, Ball quotes from an article by Berthold-Georg Englert:

"[There is a] widespread habit of ... debaters to endow the mathematical symbols of the [Schrödinger] formalism with more meaning than they have. In particular, there is a shared desire to regard the Schrödinger wave function as a physical object itself after forgetting, or refusing to accept, that it is merely a mathematical tool that we use for a description of the physical object." [19, p. 12], quoted in [4, Locations 1398–1408].

In summary: A wavefunction is not a physical entity, it is merely mathematical abstraction.

6.1.2 Source of confusion: the assumption that the 'collapse' of the wavefunction is a physical process that we might observe

Since a wavefunction is not a physical entity, it is also misleading to speak or write as if the 'collapse' of a wavefunction was a physical process that we might observe. In this connection, Freeman Dyson writes:

"Unfortunately, people writing about quantum mechanics often use the phrase 'collapse of the wave function' to describe what happens when an object is observed. This phrase gives a misleading idea that the wave function itself is a physical object. A physical object can collapse when it bumps into an obstacle. But a wave function cannot be a physical object. A wave function is a description of a probability, and a probability is a statement of ignorance. Ignorance is not a physical object, and neither is a wave function. When new knowledge displaces ignorance, the wave function does not collapse; it merely becomes irrelevant." In "The Collapse Of The Wave Function" by Freeman Dyson in [9, p. 73].

In summary: The 'collapse' of a wavefunction is a mathematical abstraction, not a physical process.

6.1.3 Source of confusion: failure to understand exactly how quantum particles may be seen to be in more than one state at the same time

What appears to be another misconception is the idea that a quantum particle can be in two mutually exclusive states *at the same time* is described by Ball thus:

"The classical idea of a state generally has an exclusive aspect to it. Macroscopic objects can be a bit of this and a bit of that—a bit rigid but somewhat flexible, or kind of reddish brown. But they can't be in mutually exclusive states: *here* and *there*, having a mass of 1 g and also of 1 kg. I can't be cycling at 20 mph at the same time as cycling at 10 mph. And my cycling jacket can't be bright yellow at the same time as being pink. It can be a mixture of both, but it can't be all yellow and all pink. This seems common sense.

"So it's understandable that, when we hear that quantum particles can be in more than one state at the same time, we struggle to see what that could mean, and we start to talk about quantum weirdness—or figure that we're too plain dumb to comprehend quantum mechanics." [4, Location 672].

And later he writes:

"This 'two (or more) states at once' is called a superposition. The terminology conjures up the image of a ghostly double exposure. But strictly speaking a superposition should be considered only as an abstract mathematical thing. The expression comes from wave mechanics: we can write the equation for a wave as the sum of equations for two or more other waves." ([4, Locations 683–690], emphasis added).

One of the pioneers of QM, Paul Dirac, provides additional clarity:

"The non-classical nature of the superposition process is brought out clearly if we consider the superposition of two states, A and B, such that there exists an observation which, when made on the system in state A, is certain to lead to one particular result, a say, and when made on the system in state B is certain to lead to some different result, b say. What will be the result of the observation when made on the system in the superposed state? The answer is that the result will be sometimes a and sometimes b, according to a probability law depending on the relative weights of A and B in the superposition process. It will never be different from both a and b. The intermediate character of the state formed by superposition thus expresses itself through the probability of a particular result for an observation being intermediate between the corresponding probabilities for the original states, not through the result itself being intermediate between the corresponding results for the original states." ([16, Locations 359–367], emphasis in the original).

From what Dirac says, it is misleading to think of superposition as resembling the superposition of waves. This is because two or more waves may exist at the same time, but in QM it is highly misleading to suggest that a quantum entity has two or more (mutually exclusive) states "at once" or "at the same time". We should not confuse a given physical state with the probability of observing that physical state.

In summary:

- The word 'superposition' in wave mechanics applies to two or more things (waves) which can exist at the same time.
- The word 'superposition' in QM is used to mean two or more things that are mutually exclusive in a given context, like the states A and B in the quote from Dirac, above.
- Although two or more states may be mutually exclusive in a given context, the probabilities of those states may be known, and the probabilities may co-exist.

There is more about this issue in Section 6.2, below.

6.1.4 Source of confusion: the idea that measurable states of quantum particles do not have particular values until we measure them

With the double-slit experiment as the implied or explicit context for discussion, another thing which can be puzzling for many people about the Copenhagen concept of superposition, is the idea that the value of some variable only comes into existence by being measured or observed:

"... we have no problem saying that [a] tennis ball was travelling at 100 mph and then I measured it. The tennis ball had the pre-existing property of a speed of 100 mph, which I could determine by measurement. We would never think of saying that it was travelling at 100 mph *because* I measured it. That wouldn't make any sense. In quantum theory, we do have to make statements like that. And then we can't help asking what it means. That's when the arguments start." [4, Locations 372–382], emphasis in the original).

And later,

"... for Bohr, all one can meaningfully say about a quantum system is contained in the Schrödinger equation. So if the maths says that we can't measure some observable quantity with more than a certain degree of precision, that quantity simply does not exist with greater precision. That is the difference between uncertainty ('I'm not sure what it is') and unknowability ('It *is* only to *this* degree'). ([4, Locations 1716–1725], emphasis in the original).

In terms of our everyday interpretation of things and events, the Bohr interpretation makes a certain amount of sense for such things as discoveries. Depending on one's use of language, a new discovery does not exist until the discovery has been made, although there is room for debate about the passage of time between the event itself and when reporters learn about it, and, from then, a further period of time before members of the public get to hear or read about it.

But more generally, the Copenhagen interpretation appears to be nonsense. For example, before we meet someone on a blind date (without any prior information), we are confident that he or she exists, and that he or she has a certain height, a certain hair colour, a certain timbre of voice, and so on, even though we do not know exactly what the values of those attributes may be. We see those values as things that exist prior to our meeting which we discover on the occasion of the meeting, not as values that spring into existence at the time we first see or hear them.

However, there is an alternative view of waves and particles that seems more in accord with what Bohr and others say about QM phenomena. This is a tentative 'tsunami' view of waves and particles, outlined in Appendix D. In brief, a subatomic wave is like a marine tsunami when it is far out to sea, with little impact on its surroundings. But, like a real tsunami when it reaches a shore, the subatomic wave changes its character when it meets any kind of detection device. When the detection device is something like a photographic plate, we see a black spot that we interpret as the effect of a particle on the plate, but it might equally well be something like a tiny 'tsunami' which has the energy to cause a chemical change in the plate.

Summary: In terms of our everyday experience, it is wrong to suppose that, in QM, quantum particles with measurable values do not exist until we measure them. But there is a better fit between that interpretation and the tentative 'tsunami' view of waves and particles described in Appendix D.

6.2 Two analogies for the concept of 'superposition' in QM

It seems that, in Section 6.1.1 (about the concept of a 'wavefunction'), in Section 6.1.2 (about the 'collapse' of a wavefunction), and in Section 6.1.3 (about 'superposition' of states), the heart of the problem appears to be surprisingly widespread misunderstandings about QM. But in Section 6.1.4 (about the idea that the value of any state of a quantum particle does not exist until it is detected or measured), the problem seems to lie with QM itself. What appears to be nonsense in terms of our everyday experience may make better sense in terms of 'tsunami' interpretation of wave-particle duality.

The two subsections that follow describe similarities between the meaning of the word 'superposition' as it is used in QM (which, as previously noted, appears to be *different* from the meaning of that word in wave mechanics) and: 1) the concept of 'syntactic class' in theoretical linguistics; and 2) the concept of 'data type' in mathematics and computing.

6.2.1 The similarity between 'superposition' in quantum mechanics and 'syntactic class' in theoretical linguistics

This section describes what seems to be a useful analogy for the idea of 'superposition' of two or more mutually exclusive states of a quantum entity (Section 6.1.3).

Figure 4 shows a set of SP-patterns that may be seen as a stochastic grammar for a very simple English-like language.²

The first SP-pattern in Figure 4, '(S s1 D #D N #N V #V #S)', represents the abstract structure of a sentence. Within that SP-pattern: 'D #D' may be seen as a 'slot', 'space', or 'variable', for a word of the grammatical category 'determiner' (words like 'the', 'a', 'two', and so on); 'N #N' may be seen as a variable for a word of the category 'noun'; and 'V #V' may be seen as a variable for a word of the category 'verb'.

 $^{^2\}mathrm{A}$ 'stochastic' grammar is a grammar in which the frequency of usage, or probability, of each rule, has a role to play.

(S s1 D #D N #N V #V #S)*750
(D d1 the #D)*600
(D d2 this #D)*150
(N n1 dog #N)*400
(N n2 cat #N)*350
(V v1 walks #V)*500
(V v2 runs #V)*250

Figure 4: SP-patterns representing grammatical structures, as discussed in the text. The number after each SP-pattern, preceded by a '*', is a supposed frequency of occurrence in some imaginary text.

The remaining SP-patterns in Figure 4 represent words, each one with its main syntactic class:

- The SP-patterns '(D d1 the #D)' and '(D d2 this #D)' are words of the syntactic class 'determiner' (marked by 'D' and '#D' in the SP-pattern).
- The SP-patterns '(N n1 dog #N)' and '(N n2 cat #N)' are words of the category 'noun' (marked by 'N' and '#N' in the SP-pattern).
- The SP-patterns '(V v1 walks #V)' and '(V v2 runs #V)' represent words of the category 'verb' (marked by 'V' and '#V' in the SP-pattern).

When the SPCM is run with the New SP-pattern, '(this dog runs)', and with the SP-patterns in Figure 4 as Old SP-patterns, the best SPMA created by the program is the one shown in Figure 5. Here, "best" means that the SPMA in the figure is the one which, via an encoding (as described in [60, Section 4.1] and [58, Section 3.5]), yields the greatest compression of the New SP-pattern. Overall, the SPMA may be seen as a parsing of the sentence '(this dog runs)' in terms of its grammatical constituents.

0	this			dog				runs			0
				1				Ι			
1		N	n1	dog	#N			Ι			1
	1				Ι						
2 S s1 D	I	#D N			#N	V		Ι	#V	#S	2
I	I	1						Ι	Ι		
3 D	d2 this	#D						Ι	Ι		3
								Ι	Ι		
4						V	v2	runs	#V		4

Figure 5: The best SPMA created by the SPCM using the New SP-pattern '(this dog runs)' and the Old SP-patterns shown in Figure 4.

This example shows how, in a parsing of a simple sentence, each of the variables 'D #D', 'N #N', and 'V #V', may take on appropriate values, depending on the sentence being parsed.

In summary, a superposition in QM is similar to a syntactic class in four respects:

- 'Superposition' in QM and 'syntactic class' in linguistics are both abstractions, without any corresponding physical structure in the world;
- They both represent two or more values that are *alternatives* in each of one or more contexts;
- With both superposition and syntactic class, there are situations where one value is selected out of the two or more values in the superposition or syntactic class. In QM, this happens when a 'particle' is detected or 'measured'. With syntactic classes, this happens when a 'variable' is assigned a value from the relevant syntactic class, either during the interpretation of a sentence or during the process of constructing a new sentence.
- In a superposition and in a syntactic class in a stochastic grammar, there is a frequency or probability associated with each rule. Non-stochastic grammars in linguistics leave out this refinement, but in effect that means that all the values of any given syntactic class have the same frequency.

With regard to the idea that, in QM, quantum particles may be seen to be in more than one state *at the same time* (Section 6.1.3), the analogy between a syntactic class and superposition in QM seems to provide an answer:

- 1. As before, it is clear that two or more physical waves may occur at the same time.
- 2. But within a given context within any sentence that conforms to the grammar shown in Figure 4, nouns like 'dog' and 'cat', emphatically, *do not* occur *at the same time*, and likewise for determiners like 'the' and 'this', and for verbs like 'runs' and 'walks'.
- 3. However, within an SP-grammar like that shown in Figure 4, the class 'noun' contains 'dog' and 'cat' *at the same time*, each with a frequency value. Likewise for the class 'determiner' (words like 'the' and 'this'), and for the class 'verb' (words like 'runs' and 'walks').

It seems that the puzzlement that many people experience in hearing that two or more quantum attributes may occur at the same time is because of confusion between, on the one hand, items 1 and 2 above, (which do not reflect QM usage), and, on the other hand, item 3 (which does reflect QM usage).

6.2.2 The similarity between 'superposition' in quantum mechanics and 'data type' in mathematics and computing

As with the concept of superposition in QM and its similarity with the concept of syntactic class, there is a similarity between the concept of superposition and the concept of a 'data type' in mathematics and computing. As before, it appears that they are similar in four respects:

- Both 'superposition' in QM and 'data type' in mathematics or computing are abstract constructs without any corresponding physical structure;
- A superposition and a 'data type' both represent a range of possible values that are *alternatives* in each of one or more contexts;
- With both superposition and data type, there are situations where one value is selected out of the two or more values in the superposition or data type;
- As with superposition, probabilistic programming on computers assigns a probability or frequency to each of the values of each data type. Where no such probabilities are assigned, we may assume that they are all the same.

Further observations are the same *mutatis mutandis* as in Section 6.2.1.

In general, the names 'superposition' (in QM), 'syntactic class' (in theoretical linguistics), and 'data type' (in mathematics and computing), may be seen to be three different terms for one concept.

6.3 Summary of main points in Section 6 about the QM concept of 'superposition'

In summary, the main points that have been made about the QM concept of 'superposition' are:

- A wavefunction is not a physical entity, it is merely mathematical abstraction (Section 6.1.1).
- The 'collapse of a wavefunction' is not a physical process, it is a mathematical abstraction (Section 6.1.2).
- Contrary to how the concept of 'superposition' in QM is often misunderstood, 'superposition' in QM applies to two or more things that are *mutually exclusive* in a given context (Section 6.1.3). They occur at the same time only in the sense that, in linguistics, 'dog' and 'cat' are expressed as members of the class 'noun', at the same time, each with a probability.

- In terms of our everyday experience, it is wrong to suppose that, in QM, quantum particles with measurable values do not exist until we measure them. But there is a better fit between that interpretation and the tentative 'tsunami' view of wave-particle duality outlined in Appendix D (Section 6.1.4).
- There are several points of similarity between the concept of 'superposition' in QM and: 1) the concept of 'syntactic class' in theoretical linguistics; and 2) the concept of 'data type' in mathematics and computing (Section 6.2). They may be seen as three different terms for one concept.

6.4 Quantum computing

This section considers possible implications of what has been said in this main section for concepts associated with quantum computing?

The main ideas in quantum computing are described by Al-Khalili thus:

"... in 1985, Oxford physicist David Deutsch published a pioneering paper that showed how [quantum computing] might be achieved in practice. ... Deutsch's machine would operate according to quantum principles to simulate any physical process. It required a row of quantum systems that could each exist in a superposition of two states, such as atoms in superpositions of two energy levels. These quantum systems would then be entangled together to create quantum logic gates that would be made to perform certain operations.

The basic idea is that of the 'quantum bit' or qubit. In a normal digital computer, the basic component is the 'bit', a switch that can be in either of two positions: off or on. These are denoted by the binary symbols of 0 and 1. However, if a quantum system, such as an atom, is used then it could exist in the two states at once. A qubit can thus be both off and on at the same time, just as long as it can be kept isolated from its environment.

Of course a single qubit is not very useful. But if we entangle two or more qubits we can start to see the power of such a set-up. Consider the information content of three classical bits. Each can be either 0 or 1 and so there are eight different combinations of the three (000, 001, 010, 100, 011, 101, 110, 111). But just three entangled qubits allow us to store all eight combinations at once! Each of the three digits is both a 1 and a 0 at the same time.

Adding a fourth qubit would give us 16 combinations and a fifth, 32 and so on. The amount of information stored increases exponentially

(as 2^N , where N is the number of qubits). Now imagine carrying out operations in the same way that we would with classical bits. We would be able to perform 2^N computations at once, the ultimate in parallel processing. Certain problems that might take a normal supercomputer years to solve could be cracked in a fraction of a second. [1, locations 3421-3433].

In the light of what has been said about superposition in Section 6, it seems necessary to view a qubit as an abstraction *that describes '0' and '1' as alternatives in a given context*. Contrary to the way qubits are often described for non-specialist readers, they are *not* versions of classical bits that can, in some mysterious way, be a "ghostly double exposure" of '0' and '1' at the same time.

6.4.1 Syntactic classes, data types, and quantum computing

If the parallels described in Sections 6.2.1 and 6.2.2 are accepted, then in the SPpattern 'S s1 D #D N #N V #V #S' in Figure 4, the syntactic variable 'D #D' is like a superposition of the SP-patterns '(D d1 the #D)' and '(D d2 this #D)', the syntactic variable 'N #N' is like a superposition of the SP-patterns '(N n1 dog #N)' and '(N n2 cat #N)', and the syntactic variable 'V #V' is like a superposition of the SP-patterns '(V v1 walks #V)' and '(V v2 runs #V)'.

Viewed in that way, the whole grammatical structure may be seen as having the same general form as the three entangled qubits in the quote from Al-Khalili near the beginning of Section 6.4 above:

- The three entangled qubits is equivalent to eight possible three-digit patterns: (000, 001, 010, 100, 011, 101, 110, 111).
- In much the same way, the SP-pattern 'S s1 D #D N #N V #V #S' in the SPgrammar in Figure 4, together with other SP-pattern in the SP-grammar, is equivalent to eight sentences: 'texttthe dog walks', 'texttthe dog runs', 'texttthe cat walks', 'texttthis dog walks', 'texttthe cat runs', 'texttthis dog runs', 'texttthis cat walks', and 'texttthis cat runs',

6.4.2 Quantum computing in comparison with non-quantum parallel processing

What is said about syntactic classes in Sections 6.2.1 and 6.4.1 suggests that, with a conventional computer working on a linguistic application, or something with similar characteristics, non-quantum parallel processing (NQPP) may be applied very simply by treating each invocation of a syntactic class or similar set of alternatives as an opportunity to process all the members of the class in parallel.

Since superposition also resembles the concept of a 'data type' in mathematics or computing (Section 6.2.2), much the same may be said about qubits in quantum computing.

Thus, we may suppose that the anticipated speedup in any quantum computer would be largely because the '0' and '1' in each qubit would be processed in parallel, in essentially the same way as an ordinary parallel-processing computer, but with different hardware technologies.

Possibilities like that are discussed quite fully by Ball in a chapter in [4] headed "Quantum computers don't necessarily perform 'many calculations at once'" ([4, Locations 3010], to [4, Location 3098]).

Without prejudging the outcomes of the several debates described by Ball, there seem to be good reasons to devote as much attention to the development of NQPP as are devoted to the development of quantum computing:

- The range of views described by Ball, mentioned above, demonstrate many uncertainties in current thinking about quantum computing.
- The undoubted technical difficulties in making quantum computers work: "Just as in a classical computer, the 1s and 0s of the input to a quantum algorithm are marshalled into binary digits encoding solutions. The catch is that superpositions are generally very 'delicate'. They get easily disrupted by disturbances from the surrounding environment, particularly the randomizing effects of heat. ... this doesn't really mean—as is often implied—that superpositions are destroyed, but rather that the quantum coherence spreads into the environment, so that the original system decoheres." [4, Location 2822].
- It appears to be generally accepted that: "There isn't a straightforward way of making use of what QM has to offer, and designing good quantum algorithms is a very difficult task." [4, Location 3084]
- A report in the *Communications of the ACM* describes how a senior honors student, at the University of Texas at Austin, "discovered an algorithm that showed classical computers can indeed tackle predictive recommendations at a speed previously thought possible only with quantum computers." [23, p. 15].
- A paper by Mikhail Dyakonov [17] argues that the astronomically-large number of "degrees of freedom" in quantum computing means that, in answer to the question "When will we have a quantum computer?" in the title of the paper, "As soon as physicists and engineers learn to control this number of degrees of freedom, which means—never!" [17, p. 4].

• It seems possible that any speed advantage of a quantum computer compared with an ordinary parallel-processing computer, would be due to the underlying hardware rather than QM concepts as such.

6.4.3 Parallel processing and the DONSVIC principle

This subsection describes how the efficiency of NQPP may relate to the DONSVIC principle—the Discovery of Natural Structures Via Information Compression—described in [60, Section 5.2].

Although there are shortcomings in unsupervised learning in the current version of the SPCM [60, Section 3.3], unsupervised learning is an important part of the thinking behind the SPTI, and it is anticipated that a mature version of unsupervised learning in the SPCM will conform to the DONSVIC principle.

Another way of expressing this is to say that, if a body of information, \mathbf{I} , is compressed by means of a well-developed version of unsupervised learning, the result will be, $\mathbf{I}_{\mathbf{C}}$, a compressed version of \mathbf{I} which is at or near the maximum possible for \mathbf{I} . And in view of the effectiveness of the human brain in compressing information, it is likely also to have a structure which people regard as 'natural'.

Here, 'natural' structures include things that we naturally recognise as coherent entities—such as words like 'dog' and 'cat', and objects like real dogs and cats—and groupings that we naturally recognise as being alternatives in a given context—like the syntactic classes in the SP-grammar in Figure 4, or alternative candidates for election as an MP in a given constituency, and so on. In short, a mature version of unsupervised learning in the SPTI is likely to yield data that conform to the DONSVIC principle.

Let us now return to a central point in Section 6.4.2, that NQPP may be applied very simply by treating each invocation of a syntactic class, or a similar set of alternatives, as an opportunity to process all the members of the class in parallel. The suggestion here is that, with a body of knowledge, $\mathbf{I}_{\mathbf{C}}$, that conforms to the DONSVIC principle, NQPP that takes advantage of sets of alternatives within $\mathbf{I}_{\mathbf{C}}$ is likely to be very much more efficient than any kind of bit-level NQPP. Tentatively, this is because, with alternatives appearing as relatively large 'chunks' of information within $\mathbf{I}_{\mathbf{C}}$, there would be very much less switching between alternatives than with bit-level parallelism.

Whether or not the same arguments would apply to a comparison of NQPP with bit-level quantum computing is not clear. All one can say with any confidence is that there is potential for NQPP applied to data that conforms to the DONSVIC principle has the potential to be at least as efficient as bit-level quantum computing.

7 Nonlocality, entanglement, SPMA, and discontinuous dependencies

The interrelated concepts of 'superposition' (discussed in Section 6), 'nonlocality' and 'entanglement' are described by Al-Khalili thus:

"[Superposition] states that a quantum particle can be in a combination of two or more (mutually exclusive) states at the same time, while [nonlocality] says that two quantum particles (or two separate parts of the spread-out wavefunction of the same particle) can somehow remain in touch with each other however far apart they are. I will now combine these two ideas together in order to introduce a third quantum concept.

"In quantum mechanics, the idea of two dice remaining in (nonlocal) contact with each other how ever far apart they are is known as entanglement." [1, 1225–1234].

There is little doubt that the phenomena of 'nonlocality' and 'entanglement' are genuine features of the world and not merely some 'weirdness' in QM which may, at some stage, be explained away:

"Today, quantum nonlocality and entanglement are no longer the subject of philosophical debate. They are accepted as crucial features of the quantum world. Indeed, entanglement of many particles could lead to the development of a whole new technology not even dreamed of by the quantum pioneers." [1, location 1274].

7.1 A potential analogy for QM-nonlocality and QM-entanglement in the processing of natural language

As with QM-superposition, it seems that there is a potentially useful analogy for QM-nonlocality and QM-entanglement in the processing of natural language (Section 6.2.1).

The SPMA shown in Figure 7 provides an example (with simplifications of some of the details of English grammar). Here, the pattern 't h e p l u m s a r e r i p e' is identified as a sentence (defined by the SP-pattern 'S ... #S' in row 7), and parsed into constituents such as a noun phrase ('NP ... #NP' in row 4), a verb phrase ('V ... #V' in row 6), and so on.

For present purposes, the key point of interest is that, within sentences like 't h e p l u m s a r e r i p e', there is a syntactic 'dependency' between the 'subject' at the beginning (which is the two-word noun-phrase 't h e p l u m

s') and the verb-phrase later in the sentence (which is the two-word phrase 'a r e r i p e').

The rule here is that, in English at least, if the 'subject' noun-phrase is plural then the main or only verb-phrase must be plural, and if the subject noun-phrase is singular then the main verb-phrase must be singular.

Most natural languages have dependencies like that, such as for example, gender dependencies in French, which may cut across number dependencies (for more discussion, see [58, Section 5.4]). An interesting example, which may be modelled with the SPCM, is the intricate pattern of dependencies in English auxiliary verbs ([60, Section 8.2] and [58, Section 5.5]).

This kind of dependency is often described as 'discontinuous' because, in a sentence like in *the plums, that grew large this year, are ripe*, the plural dependency, *the plums ... are ripe* can jump over intervening structure (*that grew large this year* in this example), and there appears to be no limit on how big that intervening structure may be.

Amongst the several ways in which discontinuous dependencies may be represented in AI systems, one of the simplest is via an SP-pattern within an SPMA, like the SP-patten in row 8 in Figure 7. Here, the SP-symbol 'Npl' (mnemonic for 'plural noun-phrase') is aligned with a matching SP-symbol within the SPpattern for the subject noun-phrase, 'NP ... #NP' in row 4, and the symbol 'Vpl' (mnemonic for 'plural verb-phrase') is aligned with a matching symbol within the SP-pattern for the main verb-phrase, 'VP ... #VP in row 6.

The fact that the SP-symbols 'NPp' and 'VPp' both appear in one SP-pattern (in row 8) is what marks the dependency between the subject noun-phrase and the main verb-phrase.

This example suggests that insights gained with the SPTI may have traction in QM. It seems possible that a dependency between, for example, two entangled electrons, such that one electron has a clockwise spin while the other electron has a counter-clockwise spin, may be understood in a manner that is similar to our understanding of the phenomenon of syntactic dependencies in natural languages. In both cases:

- There is a correlation between the two elements of the dependency.
- The dependency may bridge arbitrarily large amounts of intervening structure.
- There is a kind of 'instant' communication in the sense that, if we know one element of a dependent pair, we know immediately what the other should be. This effect is what Einstein famously called 'spooky action at a distance'.³

³ "As he declared to his friend Max Born, coining a memorable phrase, 'Physics should

The kind of instant communication just mentioned—something that has been verified in many experiments—looks like communication that is faster than the speed of light and thus incompatible with a basic principle in general relativity that nothing can travel faster than light. The next subsection describes how that apparent contradiction may be resolved.

7.2 How a non-local, entangled pair of particles may be regarded as a single entity

A suggested answer to the issue raised at the end of the preceding subsection is that what is normally construed as *two* entangled particles could equally well be seen as a single object, in the same way that the SP-pattern 'Num PL ; Npl Vpl' in row 8 of Figure 7 is a single object containing the two significant SP-symbols, 'Npl' and 'Vpl'. In this case, there is no need for any communication at all, spooky or otherwise, because if we have a full knowledge of an SP-pattern, we know its contents.

Ball makes essentially the same point in a chapter in [4] headed "There is no 'spooky action at a distance'" [4, location 1973]. Without attempting to discuss all the arguments and counter-arguments that Ball considers, here is one of the more telling examples that he describes:

"Think of a pair of gloves: one left-handed, the other right-handed. If we were to post one at random to Alice in Aberdeen and the other to Bob in Beijing ..., then the moment Alice opened the parcel and found the left glove (say), she'd know that Bob's glove is right-handed. This is trivial, because the gloves had that handedness all the time they were in transit—it's just that Alice and Bob didn't know which was which until one of them looked." [4, locations 1838–1845].

In the same vein, Ball writes a little later that:

"We can't regard particle A and particle B [that are entangled] as separate entities, even though they are separated in space. As far as quantum mechanics is concerned, entanglement makes them both parts of a single object." [4, location 2026].

Here's another example. Imagine a scene in which a car is partly obscured by the trunk of a tree, with the front part visible. If we see the front part move

represent a reality in time and space, free from spooky action at a distance.'", this quote, including the quote from Einstein, is in [27, Location 8066]. The source for what Einstein said is given as "Einstein to Max Born, March 3rd, 1947, in [8, p. 155] (not in *Albert Einstein Archives*).

forwards (or backwards), we can infer instantly that the back of the car will be moving in the same direction and at the same speed. Of course, it could be a stage magician's car that does something different—so, for that reason, the inference is probabilistic. But in this case the probabilities strongly favour the normal interpretation.

Since this kind of scene is very familiar, it would be strange indeed if people were to speak of nonlocality and entanglement between the front and back of a car! Perhaps we'll eventually drop that kind of language when speaking or writing about entangled quantum particles.

Similar things can be said about left and right brackets as they are normally used in text. Although they are normally separated by a body of text, which can be quite variable in its size, we think of them as belonging to a single entity, which leads to expectations, such as a left round bracket being followed by a right round bracket, or a left square bracket being followed by a right square bracket, and so on.

8 Our Mathematical Universe and the SPTI

This section is about ideas described by physicist Max Tegmark in his book *Our Mathematical Universe* [51], and the suggested relevance of the SPTI.

In the book, Tegmark argues for the "... crazy-sounding belief of mine that our physical world not only is *described* by mathematics, but that it *is* mathematics, making us self-aware parts of a giant mathematical object" (the 'Mathematical Universe Hypothesis', MUH), ([51, p. 6], emphasis in the original).

In connection with that idea, Tegmark describes several concepts:

- 'Our universe': "The part of physical reality we can in principle observe; quantum complications aside, this is the spherical region of space from which light has had time to reach us during the 14 billion years since our Big Bang." [51, p. 138];
- 'Parallel universe': "A part of physical reality that can in principle be observed from somewhere else but not from here—parallel universes are not a theory, but a prediction of certain theories." (*ibid.*);
- 'Multiverse': "a collection of universes" (*ibid.*);
- 'Level I multiverse': "Distant regions of space that are currently but not forever unobservable; they have the same effective laws of physics but may have different histories" (*ibid.*);

- 'Level II multiverse': "Distant regions of space that are forever unobservable because space between here and there keeps inflating; they obey the same fundamental laws of physics, but their effective laws of physics may differ." [51, pp. 138–139];
- 'Level III multiverse': "Different parts of quantum Hilbert space ...; same diversity as Level II." [51, pp. 139];
- 'Level IV multiverse': "All mathematical structures ... corresponding to different fundamental laws of physics" (*ibid.*);

And Tegmark describes a set of ideas with approval [51, pp. 185–191] that were originated in 1957 by Hugh Everett III in his PhD thesis at Princeton University [20].

In brief, Everett proposed a radical alternative to Niels Bohr's Copenhagen interpretation of QM. Instead of supposing a 'collapse' of the wavefunction whenever someone makes an 'observation' of a quantum entity, Everett supposed that the wavefunction would never collapse, regardless of any 'observation', and the alternatives that it describes (such as heads or tails for a coin flip) would, in effect, mean a splitting of the universe into two, corresponding to the heads or tails alternatives. Tegmark writes:

"... parallel-universe splitting is happening constantly, making the number of quantum parallel universes truly dizzying. Since such splitting has been going on ever since our Big Bang, pretty much any version of history that you can imagine has actually played out in a quantum parallel universe, as long as it doesn't violate any physical laws. This makes vastly more parallel universes than there are grains of sand in our Universe. In summary, Everett showed that if the wavefunction never collapses, then the familiar reality that we perceive is merely the tip of an ontological iceberg, constituting a minuscule part of the true quantum reality." [51, p. 190].

And further:

"Let's call the quantum parallel universes that Everett discovered Level III parallel universes, and the collection of all of them the Level III multiverse. Where are these parallel universes? Whereas the Level I and Level II kinds are far away in our good old three-dimensional space, the Level III ones can be right here as far as these three dimensions are concerned, but separated from us in what mathematicians call Hilbert space, an abstract space with infinitely many dimensions where the wavefunction lives." [51, p. 190].

and "Everett's version of QM first began to get popularized by the famous quantumgravity theorist Bryce DeWitt, who called it the *Many Worlds* interpretation—a name that stuck." [51, p. 190].

What about Level IV multiverses? Tegmark suggests that "mathematical existence and physical existence are equivalent, so that *all* structures that exist mathematically exist physically as well." ([51, p. 321], emphasis in the original) and "there's a fourth level of parallel universes that's vastly larger than the [other three levels], corresponding to different mathematical structures." (*ibid.*).

Tegmark also writes: "According to the CUH [Computible Universe Hypothesis], the mathematical structure that is our physical reality has the attractive property of being computable and hence well defined in the strong sense that all its relations can be computed. There would thus be no physical aspects of our Universe that are uncomputable/undecidable, eliminating the concern that the work of Church, Turing and Gödel somehow makes our world incomplete or inconsistent." [51, p. 333]. In other words, the Level IV multiverse would only contain mathematics that is, metaphorically, kosher.

The subsections that follow describe how the MUH ideas may be interpreted from other viewpoints, including perspectives from the SPTI.

8.1 Fudging the distinction between 'abstract' and 'physical'

Many people, including many physicists, are likely to agree with Tegmark that the MUH is "crazy-sounding", perhaps with the rider that it is indeed crazy.

One apparent problem is that MUH makes no provision for the way in which mathematics is often used to abstract away from any physical reality, so that an abstraction like '4' may be used to say something useful about physical entities such as apples, people, cups of tea, and so on.

What is the 'reality' of 4, 4×5 , 4^8 , 4!, and the many other abstract entities in mathematics? The belief that they have some kind of 'real' existence would be much like the 'Platonism' view that mathematical entities are "numinous and transcendent entities, existing independently of both the phenomena they order and the human mind that perceives them" [25, pp. 95–96]. In keeping with that idea, Tegmark writes:

"[Each mathematical structure] describes a physically real universe just a different one from the one we happen to inhabit. This can be viewed as a form of radical Platonism, asserting that all the mathematical structures in Plato's 'realm of ideas' exist 'out there' in a physical sense." [51, p. 320–321].
Of course, Plato was a great thinker, but like all great thinkers, he could make mistakes. And he would probably have been the first to admit that he had not entirely 'nailed' the fundamental nature of mathematics. With the passage of time and a great deal more thought, and with the benefit of insights from researchers such as Claude Shannon [47] and Ray Solomonoff [48, 49], we can do better.

Of course, we are free to suppose that abstract structures in mathematics are themselves 'physically real' entities. But this makes nonsense of the ordinary meanings of words like 'abstract' and 'physical'. Those ordinary meanings are rooted in ideas which are in constant use everywhere: with unary arithmetic, the shepherd may count his sheep with a mark on paper or his hand for each one, in essentially the same way that a prisoner may keep track of days that have passed with marks on the wall, and so on through the many uses of numbers. From those fundamentals is built the whole abstract structure of number theory, none of which says anything about numbers of sheep, days that have passed, apples, people, and so on.

There seems to be no good reason for fudging that longstanding and extremely useful distinction between a number and what it applies to, except to salvage the extraordinary idea that mathematics can, by some obscure kind of magic, jump from being a good means of abstracting away from reality to being a reality itself.

8.2 A psychological and biological perspective

An alternative to Tegmark's view of mathematics is a psychological and biological view which derives from the SP programme of research. Here's a summary:

- Evidence for the importance of IC in HLPC. There is now a substantial body of evidence for the importance of IC, and more specifically ICMUP, in HLPC (Appendix C.3).
- *IC and natural selection*. It is clear that, for many animals including humans, IC has been and remains an important driver for natural selection (Appendix C.3).
- Mathematics, human brains, and IC. Since mathematics is the product of human brains, and is an aid to human thinking, it should not be surprising to find that mathematics, in exhibiting features of ICMUP [70, Section 5], chimes with the way in which the SPMA concept (a generalised version of ICMUP [70, Section 5.7]) exhibits versatility in modelling several aspects of HLPC (item 12 in Appendix A and [60, 58]).

In short, the SP/SPTI perspective on mathematics has deep roots in human psychology and human biology, things that are missing from the MUH.

8.3 Computability

The idea that all mathematical structures in the Computable Universe should be computable means that most AI algorithms, and most computer models of human cognition, would be excluded. This is because most of AI and the modelling of human cognition is about the use of heuristic techniques to find reasonably good approximate answers to problems that are are too complex to be computable (see, for example, [69, Section 2.2.2]). This seems to rule out any kind of intelligent being, including ourselves, in the Computable Universe, so that we cannot be "self-aware parts of a giant mathematical object" (Section 8).

8.4 What about compression of information?

With or without the SP Theory, it is widely accepted that, in the development of any kind of scientific theory, we should steer clear of things that increase complexity rather than reduce it. In those terms, the mind-boggling complexity of creating a whole new universe for each dichotomy that may arise, however small it may be, and this from the beginning of the world (Section 8), seems to be entirely wrong.

It is true that the SPCM, which works entirely via the compression of information, can achieve the paradoxical effect of decompressing information (item 11 in Appendix A). But that would normally be when the system is used to retrieve the original form of information that had previously been compressed. It is quite different from the colossal redundancy that would be generated in the Many Worlds view of QM. And that colossal redundancy is totally at odds with the widely-accepted idea that scientific theories should, as far as possible, conform to Ockham's razor.

8.5 What counts as an 'observation'?

Another feature of the Many Worlds interpretation of QM which seems to be unsatisfactory is the difficulty of establishing what counts as an 'observation' that leads to a splitting of the universe. In the quote above (from [51, p. 190]), Tegmark says that that kind of splitting "has been going on ever since our Big Bang". Since, for most of that time, people have not existed, most of the observations that led to the many splittings must have been made by non-human creatures, including the multitude of single-celled organisms. Since all such creatures are sensitive to their environments, they must be making 'observations', and since the sensitivities of most organisms are essentially continuous across several dimensions (temperature, sound, vision, etc), there would be many many quadrillions of 'observations' every microsecond since the beginnings of life on Earth. Since, in the Many Worlds interpretation of QM, each such observation would lead to a splitting of the universe, any belief one may have had in the Many Worlds interpretation of QM, is likely to be strained to breaking point.

9 Conclusion

This paper first introduces the *SP System* (SPTI), meaning the *SP Theory of Intelligence* and its realisation in the *SP Computer Model*. The SPTI is described in Appendix A, which itself contains pointers to where further information may be found.

The main aim of the paper is to explore the potential of the SPTI in science.

Section 2 describes briefly some of the potential benefits, and Section 3 outlines reasons why mathematics may be integrated with the SPTI, and potential benefits for science of such an integration. Subsections that follow summarise other sections of the paper.

9.1 Infinity (Section 5)

For some time there has been disquiet amongst scientists, especially physicists, about the way in which mathematics can all too easily suggest that something is infinitely big or infinitely small. Of course it is possible to simply ignore what the mathematics implies, but it would be more satisfactory if the 'disquiet' were to be backed by some good principles.

With examples from the SPCM, it is argued that the SPTI provides principles that suggest why the kind of recursion that is needed for inferences of infinity, would in practice be reined in.

9.2 Superposition (Section 6)

This section describes some apparent misconceptions about the concept of 'superposition' in QM, how they may be resolved, with examples from the SPCM.

Topics include: misconceptions about the concepts of 'wavefunction' and the 'collapse' of the wavefunction; how two or more things that are mutually-exclusive in a given context can be seen to occur *at the same time*; how a tentative 'tsunami' view of wave-particle duality can help to make sense of the idea that an entity or attribute does not exist until it is detected or measured; how the concept of 'superposition' in QM has points of similarity with the concept of 'syntactic class' in theoretical linguistics, and with the concept of 'data type' in mathematics or computing.

Section 6.4 describes how the topics just mentioned suggest that 'quantum computing' is not based on QM itself but is largely about new technologies for

achieving parallel processing—which should be evaluated in comparison with other technologies for parallel processing.

9.3 Nonlocality and entanglement (Section 7)

With an example from the SPCM it is argued that there are similarities between the QM concept of entanglement (with nonlocality) and the phenomenon of discontinuous dependencies in natural language. In both cases: 1) there is a correlation between two entities; 2) the relationship may bridge arbitrarily large amounts of intervening structure; and 3) because the entangled entities are, in effect, a single entity, all its attributes may be known at once, without the need for transmission, and without the need for "spooky action at a distance."

The concept of SPMA in the SPCM provides an effective way of representing two or more entangled entities that may bridge intervening structures of any size.

9.4 Our Mathematical Universe (Section 8)

This section argues against Tegmark's "... crazy-sounding belief of mine that our physical world not only is described by mathematics, but that it is mathematics, making us self-aware parts of a giant mathematical object" (the MUH) [51, p. 6], and the Many-Worlds interpretation of QM proposed by Hugh Everett III [20].

In brief, the arguments are that: 1) merging the idea of a mathematical description with the thing it describes confounds the very useful distinction between abstract and physical; 2) the SPTI provides a psychological and biological foundation for mathematics which is missing from the MUH perspective; 3) because many models of AI or human cognition are non-computable and require heuristic methods to achieve acceptable results, and because Tegmark's MUH perspective requires everything to be computable, there seems to be no place in it for AI or for models of human cognition; 4) the Many Worlds interpretation of QM is, to a great degree, in conflict with Ockham's razor.

9.5 Ways forward

As suggested in Appendix A.9, the creation of a first version of the SP Machine will provide many opportunities for other researchers, individually or in groups, to bring the SP Machine and the SPTI to maturity, as described in [39].

With increasing maturity of the SP Machine, there will be increasing opportunities for it to be applied in science, as described in this paper.

Data and software accessibility

This article has no additional data.

The SP Computer Model is available as 'SP71', under 'Gerry Wolff' in Code Ocean (codeocean.com/dashboard).

Competing interests

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Appendices

A Outline of the SP System

[This appendix is adapted from the open-access paper [73, Appendix A].]

This appendix introduces the *SP System* (SPTI), meaning the *SP Theory of Intelligence* and its realisation in the *SP Computer Model*. More detail may be found in the paper [60], and there is a much fuller account of the system in the book [58].

The SPTI is conceived as a brain-like system as shown in Figure 6, with *New* information (green) coming in via the senses (eyes and ears in the figure), and with some or all of that information compressed and stored as *Old* information (red), in the brain.



Figure 6: Schematic representation of the SPTI from an 'input' perspective. Reproduced, with permission, from Figure 1 in [60].

The strengths and potential of the SPTI are summarised in Appendix B.

A.1 IC in the SPTI

The prominent role for IC in the SPTI has three main inspirations, summarised in Appendix C.3.

In the SPTI, all IC is achieved via ICMUP (Appendix C.2) in two subsystems: SPMA (Appendix A.3) and unsupervised learning (Appendix A.4).

A.2 SP-patterns and SP-symbols

In the SPTI, all information is represented by *SP-patterns*, where an SP-pattern is array of *SP-symbols* in one or two dimensions. Here, an SP-symbol is simply a mark from an alphabet of alternatives that can be matched in a yes/no manner with any other SP-symbol.

At present, the SPCM works only with one-dimensional SP-patterns but it is envisaged that, at some stage, the SPCM will be generalised to work with twodimensional SP-patterns as well as one-dimensional SP-patterns. This should open up the system for the representation and processing of diagrams and pictures, and, as described in [61, Sections 6.1 and 6.2], structures in three dimensions.

A.3 SP-multiple-alignment

A major discovery in the SP programme of research is the concept of *SP-multiple-alignment* (SPMA), described in outline here. The SPMA concept is largely responsible for the strengths and potential of the SPTI as summarised in Appendix B.

The SPMA concept in the SPTI has been borrowed and adapted from the concept of 'multiple sequence alignment' in bioinformatics [60, Section 4]. An example of an SPMA is shown in Figure 7.



Figure 7: The best SPMA created by the SPCM that achieves the effect of parsing a sentence ('t h e p l u m s a r e r i p e') into its parts and sub-parts, as described in the text.

A.3.1 How SP-multiple-alignments are created

Here is a summary of how SP-multiple-alignments like the one shown in the figure are formed:

- 1. At the beginning of processing, the SPCM has a store of Old SP-patterns including those shown in rows 1 to 9 (one SP-pattern per row), and many others. When the SPCM is more fully developed, those Old SP-patterns would have been learned from raw data as outlined in Appendix A.4, but for now they are supplied to the program by the user.
- 2. The next step is to read in the New SP-pattern, 't h e p l u m s a r e r i p e'.
- 3. Then the program searches for 'good' matches between SP-patterns, where 'good' means matches that yield relatively high levels of compression of the New SP-pattern in terms of Old SP-patterns with which it has been unified. The details of relevant calculations are given in [60, Section 4.1] and [58, Section 3.5].
- 4. As can be seen in the figure, matches are identified at early stages between (parts of) the New SP-pattern and (parts of) the Old SP-patterns 'D 17 t h e #D', 'Nrt 6 p l u m #Nrt', 'V Vpl 11 a r e #V', and 'A 21 r i p e #A'.
- 5. Each of these matches may be seen as a partial SPMA. For example, the match between 't h e' in the New SP-pattern and the Old SP-pattern 'D 17 t h e #D' may be seen as an SPMA between the SP-pattern in row 0 and the SP-pattern in row 3.
- 6. After unification of the matching symbols, each such SPMA may be seen as a single SP-pattern. So the unification of 't h e' with 'D 17 t h e #D' yields the unified SP-pattern 'D 17 t h e #D', with exactly the same sequence of SP-symbols as the second of the two SP-patterns from which it was derived.
- 7. As processing proceeds, similar pair-wise matches and unifications eventually lead to the creation of SP-multiple-alignments like that shown in Figure 7. At every stage, all the SP-multiple-alignments that have been created are evaluated in terms of IC, and then the best SP-multiple-alignments are retained and the remainder are discarded. In this case, the overall 'winner' is the SPMA shown in Figure 7.
- 8. This process of searching for good SP-multiple-alignments in stages, with selection of good partial solutions at each stage, is an example of heuristic

search. This kind of search is necessary because there are too many possibilities for anything useful to be achieved by exhaustive search. By contrast, heuristic search can normally deliver results that are reasonably good within a reasonable time, but it cannot guarantee that the best possible solution has been found.

A.3.2 Versatility of the SP-multiple-alignment construct

As noted in the caption to Figure 7, the SPMA in the figure achieves the effect of parsing the sentence into its parts and sub-parts. But the beauty of the SPMA construct is that it is apparently largely responsible for the strengths and potential of the SPTI in several areas, summarised in Appendix B.

As noted in Appendix C.2.7, the SPMA concept is the last of seven variants of ICMUP described in Appendix C.2, and it has been shown to be a generalisation of the other six variants [72]. This generalisation is probably the main reason for the strengths and potential of the SPTI mentioned above.

For readers not already familiar with the SPTI, it is appropriate to repeat what has been said elsewhere that, bearing in mind that it would be just as bad to downplay any feature of the SPTI as would over-selling any aspect of the system, the SPMA concept promises to be as significant for our understanding of intelligence as is DNA for many aspects of biology. The SPMA concept may prove to be the 'double helix' of intelligence.

A.3.3 Origin and development of the SP-multiple-alignment construct

The SPMA concept was inspired originally from seeing the AI-potential of the bioinformatics concept of 'multiple sequence alignment'. But about 17 years of work was needed to bring the idea to its current relatively mature state of development, to explore what can be done with the concept and to write a book about it [58]. There is an outline of this work in [74, Section VI].

A.4 Unsupervised learning

"Unsupervised learning represents one of the most promising avenues for progress in AI. ... However, it is also one of the most difficult challenges facing the field. A breakthrough that allowed machines to efficiently learn in a truly unsupervised way would likely be considered one of the biggest events in AI so far, and an important waypoint on the road to AGI." Martin Ford [21, pp. 11–12], emphasis added.

As indicated above, unsupervised learning in the SPTI, described in the following two subsections, is intimately combined with processes of interpretation, as outlined in Appendix A.3.

A.4.1 Learning with a *tabula rasa*

When the SPCM is a *tabula rasa*, with no stored Old SP-patterns, the system learns by taking in New SP-patterns via it 'senses' and storing them directly as received, except that 'ID' SP-symbols are added at the beginning and end, like the SP-symbols 'A', '21', and '#A', in the SP-pattern 'A 21 r i p e #A' in row 9 of Figure 7. Those added SP-symbols provide the means of identifying and classifying SP-patterns, and they may be modified or added to by later processing.

This kind of direct learning of new information reflects the way that people may learn from a single event or experience. One experience of getting burned may teach a child to take care with hot things, and the lesson may stay with him or her for life. But we may remember quite incidental things from one experience that have no great significance in terms of pain or pleasure—such as a glimpse we may have had of a red squirrel climbing a tree.

Any or all of this one-shot learning may go into service immediately without the need for repetition, as for example: when we ask for directions in a place that we have not been to before; or how, in a discussion, we normally take account of what other people are saying.

These kinds of one-shot learning contrast sharply with learning in DNNs which requires large volumes of data and many repetitions before anything useful is learned.

"We can imagine systems that can learn by themselves without the need for huge volumes of labeled training data." Martin Ford [21, p. 12].

"... the first time you train a convolutional network you train it with thousands, possibly even millions of images of various categories." Yann LeCun [21, p. 124].

A.4.2 Learning with previously stored knowledge

Of course, with people, the closest we come to learning as a *tabular rasa* is when we are babies. At all other times, learning occurs when we already have some knowledge. In people, and in the SPTI, two kinds of things can happen:

• The New information is interpreted via SPMA in terms of the Old information, as described in Appendix A.3. The example illustrated in Figure 7 is of a purely syntactic analysis, but with the SPCM, semantic analysis is feasible too [58, Section 5.7]. • Partial matches between New and Old SP-patterns may lead to the creation of additional Old SP-patterns, as outlined next.

As an example, Figure 8 shows an SPMA between New and Old SP-patterns like the 'cat' examples mentioned earlier.

0 theblackcatwalks 0 | | | | | | | | | | | 1 < 1 the catwalks > 1

Figure 8: The best SPMA created by the SPCM with a New SP-pattern 't h e b l a c k c a t w a l k s' and an Old SP-pattern '< 1 t h e c a t w a l k s >'.

From a partial matching like this, the SPCM derives SP-patterns that reflect coherent sequences of matched and unmatched SP-symbols, and it stores the newly-created SP-patterns in its repository of Old SP-patterns, each SP-pattern with added 'ID' SP-symbols. The results in this case are the SP-patterns '< 13 t h e >', '< 19 b l a c k >', and '< 20 c a t w a l k s >'.

With this small amount of information, the SP-pattern '< 20 c a t w a l k s >' is a 'word'. But with more information such as 'c a t r u n s' or 'd o g w a l k s', the SP-patterns 'c a t' and 'w a l k s' would become separate words.

Even with simple examples like these, there is a lot of complexity in the many alternative structures that the program considers. But with the IC heuristic, the structures that are intuitively 'bad' are normally weeded out, leaving behind the structures that people regard intuitively as 'good'.

A.4.3 Unsupervised learning of SP-grammars

In the SPCM, processes like those just described provide the foundation for the unsupervised learning of *SP-grammars*, where an SP-grammar is simply a set of Old SP-patterns that is relatively good at compressing a given set of New SP-patterns.

To create 'good' SP-grammars requires step-wise processes of selection, very much like processes of that kind in the creation of 'good' SPMAs (Appendix A.3).

A.4.4 Future developments

The SPCM can learn plausible grammars from examples of an English-like artificial language but at present it cannot learn intermediate levels of structure such as phrases and clauses [60, Section 3.3]. It appears that this problem is soluble and solving it will greatly enhance the capabilities of the system.

It is envisaged that similar principles may apply to the learning of non-syntactic 'semantic' structures, and to the integration of syntax with semantics. Such developments are likely to be facilitated by generality in the way in which the SPCM represents and processes all kinds of knowledge.

With generalisation of the concept of SP-pattern to include two-dimensional SP-patterns, there is potential for the SPCM to learn 3D structures, as described in [61, Sections 6.1 and 6.2].

A.5 strengths and potential of the SPTI in AI-related functions

The AI-related strengths and potential of the SPTI are summarised in the subsections that follow. Further information may be found in [60, Sections 5 to 12], [58, Chapters 5 to 9], [66], and in other sources referenced in the subsections that follow.

A.5.1 Versatility in aspects of intelligence

As we have seen in Appendix A.4, the SPTI has strengths in the 'unsupervised' learning of new knowledge.

The SPTI also has strengths in other aspects of intelligence including: the analysis and production of natural language; pattern recognition that is robust in the face of errors in data; pattern recognition at multiple levels of abstraction; computer vision [61]; best-match and semantic kinds of information retrieval; several kinds of reasoning (next subsection); planning; and problem solving.

A.5.2 Versatility in reasoning

An aspect of intelligence where the SPTI has strengths is probabilistic reasoning in several varieties including: one-step 'deductive' reasoning; chains of reasoning; abductive reasoning; reasoning with probabilistic networks and trees; reasoning with 'rules'; nonmonotonic reasoning and reasoning with default values; Bayesian reasoning with 'explaining away'; causal reasoning; reasoning that is not supported by evidence; the inheritance of attributes in class hierarchies; and inheritance of contexts in part-whole hierarchies.

There is also potential in the system for spatial reasoning [62, Section IV-F.1], and for what-if reasoning [62, Section IV-F.2].

A.5.3 Versatility in the representation of knowledge

Within the framework of SPMA, SP-patterns may serve in the representation of several different kinds of knowledge, including: the syntax of natural languages; class-inclusion hierarchies; part-whole hierarchies; discrimination networks and trees; if-then rules; entity-relationship structures; relational tuples, and concepts in mathematics, logic, and computing, such as 'function', 'variable', 'value', 'set', and 'type definition'.

There will be more potential when the SPCM has been generalised for twodimensional SP patterns.

A.5.4 Seamless integration of diverse aspects of intelligence, and diverse kinds of knowledge, in any combination

Because diverse aspects of intelligence and diverse kinds of knowledge all flow from a single coherent and relatively simple framework—the SPMA framework it is likely that the SPCM will support the seamless integration of diverse aspects of intelligence and diverse kinds of knowledge, in any combination. It appears this is *essential* in any artificial system that aspires to the fluidity, versatility and adaptability of the human mind.

Figure 11 shows schematically how the SPTI, with SPMA centre stage, exhibits versatility and integration across diverse aspects of intelligence.



Figure 9: A schematic representation of versatility and integration in the SPTI, with SPMA centre stage.

A.6 Other potential benefits and applications

Apart from the AI-related strengths of the SPTI (Appendix A.5), there are other potential benefits and applications such as assisting with the management of big data [63], assisting with medical diagnosis [57], and others detailed on tinyurl.com/5fxhybnx.

A.7 The clear potential of the SPTI to solve 20 significant problems in AI research

The SPTI has clear potential to solve 20 significant problems in AI research [76]. All but three of those problems have been described by influential experts in AI in interviews with science writer Martin Ford, and reported by him in his book *Architects of Intelligence* [21].

A.8 SP-Neural

The SPTI has been developed primarily in terms of abstract concepts such as the SPMA construct. But a version of the SPTI called SP-Neural has also been proposed, expressed in terms of neurons and their inter-connections and intercommunications. Current thinking in that area is described in [65].

A.9 Development of an 'SP Machine'

In view of the strengths of the SPTI (Appendix A), the SPCM appears to have promise as the foundation for the development of an *SP Machine*, as described in [39].

It is envisaged that the SP Machine will feature high levels of parallel processing and a good user interface. It may serve as a vehicle for further development of the SPTI by researchers anywhere. Eventually, it should become a system with industrial strength that may be applied to the solution of many problems in government, commerce, industry, and in non-profit endeavours. A schematic view of this development is shown in Figure ??.

B Strengths of the SPTI in AI and beyond

This appendix and the following one (Appendix C.1) should not be regarded as part of the substance of the paper, described in Sections 1 to 9, excluding Section ??. It summarises information from previous publications that complement the main substance of the paper.

The strengths of the SPTI, in AI-related functions and beyond, are summarised in this appendix. Further information may be found in [60, Sections 5 to 12], in [58, Chapters 5 to 9], on the CognitionResearch.org website, and in other sources referenced below.

Most of the capabilities described in Appendix B.1 are demonstrable with the SPCM. But the word 'strengths' is also applied to aspects of the SPTI described

in Appendices B.2 and B.3 where the potential of the SPTI is clear, and to broad features of the SP research, described in Appendices B.6, B.7, and B.4.

B.1 AI-related strengths of the SPCM

What are mainly demonstrable strengths of the SPCM are summarised in this section, with indications of the few cases where a capability is not demonstrated but the potential is clear. Many examples of the workings of the SPCM are given in [58] and [60].

B.1.1 Several kinds of intelligent behaviour

The SPCM has demonstrable strengths in the following aspects of intelligence: unsupervised learning, including the discovery of segmental structures and classes of such structures; the analysis and production of natural language; pattern recognition that is robust in the face of errors in data; pattern recognition at multiple levels of abstraction; computer vision [61]; best-match and semantic kinds of information retrieval; several kinds of reasoning (next subsection); planning; and problem solving.

B.1.2 Several kinds of probabilistic reasoning

Because of the intimate relation between IC and concepts of inference and probability (see 'Algorithmic Probability Theory' developed by Solomonoff, [48, 49], [29, Chapter 4]), and owing to the fundamental role of IC in the workings of the SPTI, the system is inherently probabilistic. Accordingly, it is relatively straightforward for the SPCM to calculate absolute and relative probabilities for all aspects of intelligence exhibited by the SPCM. Details of those calculations are given in [60, Section 4.4] and [58, Section 3.7].

Kinds of reasoning that may be exhibited by the SPCM include: one-step 'deductive' reasoning; chains of reasoning; abductive reasoning; reasoning with probabilistic networks and trees; reasoning with 'rules'; nonmonotonic reasoning and reasoning with default values; Bayesian reasoning with 'explaining away'; causal reasoning; reasoning that is not supported by evidence; the inheritance of attributes in class hierarchies; and inheritance of contexts in part-whole hierarchies ([60, Section 10], [58, Chapter 7]).

There is also potential in the system for spatial reasoning [62, Section IV-F.1], and for what-if reasoning [62, Section IV-F.2].

B.1.3 The representation and processing of several kinds of AI-related knowledge

Although SP-patterns are not very expressive in themselves, they come to life in the SPMA framework within the SPCM. Within the SPMA framework, they provide relevant knowledge for each aspect of intelligence mentioned in Appendix B.1.1, for each kind of reasoning mentioned in Appendix B.1.2, and more.

More specifically, they may serve in the representation and processing of such things as: the syntax of natural languages; class-inclusion hierarchies (with or without cross classification); part-whole hierarchies; discrimination networks and trees; if-then rules; entity-relationship structures [59, Sections 3 and 4]; relational tuples (*ibid.*, Section 3), and concepts in mathematics, logic, and computing, such as 'function', 'variable', 'value', 'set', and 'type definition' ([58, Chapter 10], [64, Section 6.6.1], [67, Section 2]).

As previously noted (Section ??), the addition of two-dimensional SP patterns to the SPCM is likely to expand the capabilities of the SPTI to the representation and processing of structures in two-dimensions and three-dimensions, and the representation of procedural knowledge with parallel processing.

B.1.4 The seamless integration of diverse aspects of intelligence, and diverse kinds of knowledge, in any combination

An important additional feature of the SPCM, alongside its versatility in aspects of intelligence and diverse forms of reasoning, and its versatility in the representation and processing of diverse kinds of knowledge, is that there is clear potential for the SPCM to provide for the seamless integration of diverse aspects of intelligence and diverse forms of knowledge, in any combination. This is because those several aspects of intelligence and several kinds of knowledge all flow from a single coherent and relatively simple source: the SPMA framework.

It appears that this kind of seamless integration is *essential* in any artificial system that aspires to human-level broad intelligence.

Figure 11 shows schematically how the SPTI, with SPMA at centre stage, exhibits versatility and seamless integration.

B.1.5 How to make generalisations without over- or under-generalisation; and how to minimise the corrupting effect of 'dirty data'

The central role of IC in the workings of the SPCM (Section ??, and Appendices B.4, C.1, and B.4) provides what appears to be a sound solution to two problems with unsupervised learning: how to generalise beyond a body of data (I) without either over-generalisations (under-fitting) or under-generalisations (over-fitting);



Figure 11: A schematic representation of versatility and seamless integration in the SPTI, with the SPMA concept centre stage.

and how to learn correct forms despite the fact that I normally contains errors of various kinds, otherwise called 'dirty data'.

The proposed solution, indebted to Ray Solomonoff [48, 49], is described in [60, Section 5.3] and [58, Section 2.2.12]. In brief: compress \mathbf{I} as thoroughly as possible via unsupervised learning to yield a grammar (\mathbf{G}), and an encoding (\mathbf{E}) of \mathbf{I} in terms of \mathbf{G} . Then discard \mathbf{E} which contains all of the dirty data or most of it, and retain \mathbf{G} which provides a compact description of \mathbf{I} , including 'correct' generalisations from \mathbf{I} .

Informal tests with unsupervised learning in the SPCM, and also in the MK10 and SNPR computer models of language learning [56], suggest that these principles are sound, including the exclusion of over- and under-generalisations, and the learning of 'correct' forms without corruption by 'dirty data'.

B.2 Other potential benefits and applications of the SPTI

Apart from the foregoing distinctive features and advantages of the SPTI, it has several other potential benefits and applications. Relevant publications are outlined below, beginning with AI-related topics:

- Overview of potential benefits and applications. Several potential areas of application of the SPTI are described in [64]. The ones that relate fairly directly to AI include: best-match and semantic forms of information retrieval; the representation of knowledge, reasoning, and the semantic web.
- The development of intelligence in autonomous robots. The SPTI opens up a radically new approach to the development of intelligence in autonomous robots [62].
- Commonsense reasoning and commonsense knowledge. Largely because of research by Ernest Davis and Gary Marcus (see, for example, [15]), the challenges in this area of AI research are now better known. Preliminary work shows that the SPTI has promise in this area [68].
- An intelligent database system. The SPTI has potential in the development of an intelligent database system with several advantages compared with traditional database systems [59].
- *Medical diagnosis.* The SPTI may serve as a vehicle for medical knowledge and to assist practitioners in medical diagnosis, with potential for the automatic or semi-automatic learning of new knowledge [57].
- Natural language processing. The SPTI has strengths in the processing of natural language ([60, Section 8], [58, Chapter 5]).
- Vision, both artificial and natural. The SPTI opens up a new approach to the development of computer vision and its integration with other aspects of intelligence, and it throws light on several aspects of natural vision: [61, 73].

Other potential benefits and applications that are less closely related to AI include:

• Overview of potential benefits and applications. As mentioned above, several potential areas of application of the SPTI are described in [64]. The ones that are less directly relevant to AI include: the simplification and integration of computing systems; software engineering; the representation of knowledge IC; bioinformatics; the detection of computer viruses; and data fusion.

- Sustainability. The SPTI has clear potential for substantial reductions in the very large demands for energy of standard DNNs, and applications that need to manage huge quantities of data such as those produced by the Square Kilometre Array [71]. Where those demands are met by the burning of fossil fuels, there would be corresponding reductions in the emissions of CO₂.
- Transparency in computing. By contrast with applications with DNNs, the SPTI provides a very full and detailed audit trail of all its processing, and all its knowledge may be viewed. Also, there are reasons to believe that, when the system is more fully developed, its knowledge will normally be structured in forms that are familiar such as class-inclusion hierarchies, part-whole hierarchies, run-length coding, and more. Strengths of the SPTI in these area are described in [75].

B.3 The clear potential of the SPTI to solve 20 significant problems in AI research

Strong support for the SPTI has arisen, indirectly, from the book *Architects of Intelligence* by science writer Martin Ford [21]. To prepare for the book, he interviewed several influential experts in AI to hear their views about AI research, including opportunities and problems in the field:

"The purpose of this book is to illuminate the field of artificial intelligence as well as the opportunities and risks associated with it—by having a series of deep, wide-ranging conversations with some of the world's most prominent AI research scientists and entrepreneurs." Martin Ford [21, p. 2].

In the book, Ford reports what the AI experts say, giving them the opportunity to correct errors he may have made so that the text is a reliable description of their thinking.

This source of information has proved to be very useful in defining problems in AI research that influential experts in AI deem to be significant. This has been important from the SP perspective because, with 17 of those problems and three others—20 in all—there is clear potential for the SPTI to provide a solution.

Since these are problems with broad significance, not micro-problems of little consequence, the clear potential of the SPTI to solve them is a major result from the SP programme of research, demonstrating some of the power of the SPTI.

The paper [76] describes those 20 significant and how the SPTI may solve them. The following summary describes each of the problems briefly. Readers are invited to read [76] to see how the SPTI may solve them:

- 1. The symbolic versus sub-symbolic divide. The need to bridge the divide between symbolic and sub-symbolic kinds of knowledge and processing [76, Section 3].
- 2. *Errors in recognition*. The tendency of deep neural networks (DNNs) to make large and unexpected errors in recognition [76, Section 4].
- 3. *Natural languages.* The need to strengthen the representation and processing of natural languages, including the understanding of natural languages and the production of natural language from meanings [76, Section 5].
- 4. Unsupervised learning. Overcoming the challenges of unsupervised learning. Although DNNs can be used in unsupervised mode, they seem to lend themselves best to the supervised learning of tagged examples [76, Section 6].

It is clear that most human learning, including the learning of our first language or languages [56], is achieved via unsupervised learning, without needing tagged examples, or reinforcement learning, or a 'teacher', or other form of assistance in learning (cf. [22]).

Incidentally, a working hypothesis in the SP programme of research is that unsupervised learning can be the foundation for all other forms of learning, including learning by imitation, learning by being told, learning with rewards and punishments, and so on.

- 5. Generalisation. The need for a coherent account of generalisation, undergeneralisation (over-fitting), and over-generalisation (under-fitting). Although this is not mentioned in Ford's book [21], there is the related problem of reducing or eliminating the corrupting effect of errors in the data which is the basis of learning [76, Section 7].
- 6. One-shot learning. Unlike people, DNNs are ill-suited to the learning of usable knowledge from one exposure or experience [76, Section 8], but see references to 'CLIP' in [40, 41].
- 7. Transfer learning. Although transfer learning—incorporating old learning in newer learning—can be done to some extent with DNNs [45, Section 2.1], DNNs fail to capture the fundamental importance of transfer learning for people, or the central importance of transfer learning in the SPCM [76, Section 9].
- 8. *Reducing computational demands.* How to increase the speed of learning in AI systems, and how to reduce the demands of AI learning for large volumes of data, and for large computational resources [76, Section 10].

- 9. Transparency. Although transfer learning—incorporating old learning in newer learning—can be done to some extent with DNNs [45, Section 2.1], DNNs fail to capture the fundamental importance of transfer learning for people, or the central importance of transfer learning in the SPCM [76, Section 9].
- 10. *Probabilistic reasoning*. How to achieve probabilistic reasoning that integrates with other aspects of intelligence [76, Section 12].
- 11. *Commonsense*. The challenges of commonsense reasoning and commonsense knowledge [76, Section 13].
- 12. Top-down strategies. The need to re-balance research towards top-down strategies [76, Section 14].
- 13. *Self-driving vehicles*. How to minimise the risk of accidents with self-driving vehicles [76, Section 15].
- 14. Compositionality. By contrast with people, and the SPTI, DNNs are not well suited to the learning and representation of such compositional structures as part-whole hierarchies and class-inclusion hierarchies [76, Section 16].
- 15. Commonsense reasoning and commonsense knowledge. The challenges of commonsense reasoning and commonsense knowledge [76, Section 17].
- 16. Information compression. Establishing the key importance of IC in AI research [76, Section 18]. There is good evidence that much of HLPC may be understood as IC, and for that reason, IC is fundamental in the SPTI, including the SPCM (Section ??, Appendix B.4). By contrast, IC receives no mention in [42], and does not receive much emphasis in Schmidhuber's review of DNNs (see, for example, [44, eg, Sections 4.4, 5.6.3, 6.7]).
- 17. A biological perspective. Establishing the importance of a biological perspective in AI research [76, Section 19].
- 18. Distributed versus localist knowledge. Establishing whether or not knowledge in the brain is represented in 'distributed' or 'localist' form [76, Section 20].
- 19. Adaptation. How to bypass the limited scope for adaptation in DNNs [76, Section 21].
- 20. Catastrophic forgetting. How to eliminate the problem of catastrophic forgetting: for any one DNN, new learning wipes out or corrupts old learning [76, Section 22]. There are ways round this limitations of DNNs but they

are relatively cumbersome compared with DNNs, and require substantially more structure.

B.4 Evidence for the importance of IC in HLPC suggests that IC should be central in the SPCM

A potent idea, pioneered by Fred Attneave [2, 3], Horace Barlow [5, 6], and others, is that much of the workings of brains and nervous systems may be understood as IC. This idea has been investigated by various researchers up to the present (see, for example, [11, 12, 26]). And the importance of IC in HLPC became central in a programme of research developing computer models of the learning of a first language by children [56]. Evidence for the importance of IC in HLPC is reviewed in [69].

In connection with this research and the quest for AGI, it is of interest that, as far back as 1969, Barlow wrote:

"... the operations needed to find a less redundant code have a rather fascinating similarity to the task of answering an intelligence test, finding an appropriate scientific concept, or other exercises in the use of inductive reasoning. Thus, redundancy reduction may lead one towards understanding something about the organization of memory and intelligence, as well as pattern recognition and discrimination." [6, p. 210].

where "find[ing] a less redundant code" leads to "redundancy reduction" which means IC.

With regard to goal of developing AGI:

- Evidence for the importance of IC in HLPC has provided the motivation for making IC central in the structure and workings of the SPCM;
- In view of the same evidence, it seems clear that IC should be central in the workings of any system that aspires to AGI;
- The central role for IC in the SPCM—mediated by the concept of SPMA (Section ??)—is largely responsible for:
 - The strengths of the SPTI (Appendix B);
 - In the formation of generalisations without over-fitting or under-fitting, and in the weeding out of 'dirty data' (Appendix B.1.5);
 - And in a resolution of the apparent paradox that IC may achieve decompression as well as compression of data (Appendix B.4.1).

- In both natural and artificial systems:
 - For a given body of information, I, to be stored, IC means that a smaller store is needed. Or for a store of a given capacity, IC facilitates the storage of a larger I [69, Section 4];
 - For a given body of information, I, to be transmitted along a given channel, IC means an increase in the speed of transmission. Or for the transmission of I at a given speed, IC means a reduction in the bandwidth which is needed [69, Section 4].
- Because of the intimate relation between IC and concepts of inference and probability (Appendix B.1.2), and because of the central role of IC in the SPTI, the SPTI is intrinsically probabilistic.

Correspondingly, it is relatively straightforward to calculate absolute and relative probabilities for all aspects of intelligence exhibited by the SPTI, including several kinds of reasoning ([60, Section 4.4], [58, Section 3.7]), in keeping with the probabilistic nature of human inferences and reasoning.

B.4.1 A resolution of the apparent paradox that IC May achieve decompression as well as compression of data

It is sometimes said that IC as a central feature of HLPC conflicts with the undoubted fact that people can and do produce information as well as compress it, both in ordinary speech or writing and also in creative areas like creative writing, painting, the composition of music, and so on.

In that connection, an interesting feature of the SPCM is that SPMA processes for the analysis of New information are *exactly* the same as may be used for the production of information. For example, with natural language, processes for the production of a sentence are, without any qualification, the same as may be used for the analysis of the same sentence.

Since the SPCM works by compressing information, this feature of the SPCM looks, paradoxically, like "decompression of information by compression of information".

How the whole system works, and how this paradox may be resolved, is explained in [60, Section 4.5] and [58, Section 3.8].

There is clear potential in the SPCM for the creation of entirely new structures which may be seen as novel or creative, but not necessarily artistic. This is an aspect of the SPTI that is waiting to be explored.

B.4.2 The working hypothesis that IC may always be achieved via the matching and unification of patterns

A working hypothesis in the SP research is that all kinds of IC may be understood as ICMUP.

Although this is a "working hypothesis", there is much supporting evidence: the powerful concept of SPMA may be understood as an example of ICMUP [72]; the SPMA construct seems to underpin several aspects of intelligence (Appendix B.1.1), including several kinds of probabilistic reasoning (Appendix B.1.2); and much of mathematics, perhaps all of it, may be understood in terms of ICMUP [70].

In this research, seven main variants of ICMUP are recognised [70, Sections 5.1 to 5.7]:

- *Basic ICMUP*. Two or more instances of any pattern may be merged or 'unified' to make one instance [70, Section 5.1].
- *Chunking-with-codes.* Any pattern produced by the unification of two or more instances is termed a 'chunk'. A 'code' is a relatively short identifier for a unified chunk which may be used to represent the unified pattern in each of the locations of the original patterns [70, Section 5.2].
- Schema-plus-correction. A 'schema' is a chunk that contains one or more 'corrections' to the schema. For example, a menu in a restaurant may be seen as a schema that may be 'corrected' by a choice of starter, a choice of main course, and a choice of pudding [70, Section 5.3].
- *Run-length coding.* In run-length coding, a pattern that repeats two or more times in a sequence may be reduced to a single instance with some indication that it repeats, or perhaps with some indication of when it stops, or even more precisely, with the number of times that it repeats [70, Section 5.4].
- Class-inclusion hierarchies. Each class in a hierarchy of classes represents a group of entities that have the same attributes. Each level in the hierarchy *inherits* all the attributes from all the classes, if any, that are above it [70, Section 5.5].
- *Part-whole hierarchies.* A part-whole hierarchy is similar to a class-inclusion hierarchy but it is a hierarchy of part-whole groupings [70, Section 5.6].
- *SP-multiple-alignment*. The SPMA concept [70, Section 5.7] is described in Section ??.

The SPMA concept may be seen as a generalisation of the other six variants of *ICMUP*, as demonstrated via the SPCM in [72].

This list probably does not exhaust the possible variants of ICMUP, but they are the ones that have received most attention so far in the SP programme of research.

B.5 The SPTI provides an entirely novel perspective on the foundations of mathematics

In view of evidence for the importance of ICMUP in HLPC (Appendix C.2 and [69]), and in view of the fact that mathematics is the product of human brains and has been designed to help human thinking, it should not be surprising to find that IC is central in the structure and workings of mathematics.

In keeping with that line of thinking, the concept of ICMUP provides an entirely novel perspective on the foundations of mathematics, described in the paper [70]. It is substantially different from any of the existing 'isms' in the foundations of mathematics, but there are potential connections with structuralism [70, Section 4.4.4].

B.6 The benefits of a top-down, breadth-first research strategy with wide scope

Although the SP research strategy is not, in itself, a feature of the SPTI, the topdown, breadth-first research strategy with wide scope that is employed in the SP research is yielding breadth in the non-AI strengths of the SPTI.

Allen Newell was one of the first people to draw attention to the problems of fragmentation in cognitive science in his famous paper "You can't play 20 questions with nature and win" [34]. In that paper he exhorted researchers to tackle "a genuine slab of human behaviour" (p. 303), thus avoiding the weaknesses of micro-theories with limited scope for generalisation.

This thinking led to his book Unified Theories of Cognition [35] and a programme of research developing the Soar cognitive architecture [28], aiming for a unified theory of cognition.

These ideas chime with Pamela McCorduck's description of fragmentation in AI:

"The goals once articulated with debonair intellectual verve by AI pioneers appeared unreachable ... Subfields broke off—vision, robotics, natural language processing, machine learning, decision theory—to pursue singular goals in solitary splendor, without reference to other kinds of intelligent behaviour." [32, p. 417]. Later, she writes of "the rough shattering of AI into subfields ... and these with their own sub-subfields—that would hardly have anything to say to each other for years to come." [32, p. 424].

She adds: "Worse, for a variety of reasons, not all of them scientific, each subfield soon began settling for smaller, more modest, and measurable advances, while the grand vision held by AI's founding fathers, a general machine intelligence, seemed to contract into a negligible, probably impossible dream." [32, p. 424].

Although this quote is from 2004, much the same may be said today. There is a widespread belief that, when a satisfactory system has been developed for one aspect of intelligence, it will be possible gradually to combine it with other systems in a bottom-up strategy leading to the full generality of AGI. And that belief—much the same as what Newell criticised in [34, 35]—seems always to fail.

The reason for the persistent failure of bottom-up research strategies that are so widely pursued in AI research seems to be that any theory that works in one small part of a wide field is rarely applicable in other parts. As Dr Johnson did not say: "A bottom-up strategy in AI research is a triumph of hope over experience." As a strategy, bottom-up research looks very promising, but it has never led to anything that might yield AGI.

With these ideas and observations in mind, the SPTI has been developed via a top-down, breadth-first research strategy with an exceptionally wide scope, *aiming for a simplification and integration of observations and concepts across AI, mainstream computing, mathematics, and HLPC* (see also Appendix C.1).

The SP research strategy should help to meet the concerns of Gary Marcus and Ernest Davis: "What's missing from AI today—and likely to stay missing, until and unless the field takes a fresh approach—is broad (or "general") intelligence." [31, p. 15].

B.7 The potential benefits of a biological perspective in the development of AGI

In the same way that the SP research strategy (Appendix B.6) is yielding intrinsic strengths of the SPTI, much the same may be said about the biological perspective described in this appendix.

Since the main objective of the SP research is to develop a firm foundation for the development of AGI, it is, arguably, clear that a biological perspective is likely to be helpful, where that perspective includes knowledge of cognitive psychology and neuroscience—since humans are biological entities, and since human intelligence is a biological phenomenon. More specifically, since human intelligence is the most fully developed intelligence on the planet, a knowledge of HLPC is likely to be beneficial in guiding research towards AGI. Without that knowledge, we are unnecessarily blindfolded in our research.

With that regard, the SP research has benefited in three main ways:

- Earlier research developing computer models of the unsupervised learning of a first language, mentioned in Section ??, has provided an inspiration and foundation for the development of the SPTI.
- Recognition of the importance of IC in HLPC (Appendix B.4), which depends on studies in psychology and neuroscience.
- The author of this paper, and the main driver in developing the SPTI, has a first degree from Cambridge University in the Natural Sciences Tripos, comprising studies in experimental psychology and other biology-related sciences.

C Aspects of information compression in the SPTI

In view of the central importance of IC in the workings of the SPTI, this appendix describes aspects of IC that underpin other parts of this paper.

C.1 Simplicity and Power

[This section is based on [74, Section I-C].]

A theme of the SPTI are the twin concepts of *Simplicity* and *Power*:

- The SPTI aims to simplify and integrate observations and concepts across a broad canvass (Section 1), which means applying IC to those observations and concepts;
- IC is a central feature of the structure and workings of the SPTI (Appendix A.1), which is itself inspired by evidence for the importance of IC in HLPC [69];
- And IC may be seen as a process that increases the *Simplicity* of a body of information, **I**, whilst retaining as much as possible of the descriptive and explanatory *Power* of **I**.

Those two concepts are the reason for the name 'SP'. But it is intended that 'SP' should be treated as a name, without any need to expand the letters in the name, as with such names as 'IBM' or 'BBC'.

C.2 Information compression via the matching and unification of patterns

A working hypothesis in the SP research is that IC may, in general, be understood as a process of searching for patterns that match each other and the merging or 'unification' of patterns that are the same. The expression 'information compression via the matching and unification of patterns' may be abbreviated as 'ICMUP'.

This hypothesis may seem implausible for IC techniques with a mathematical flavour such as wavelet compression, arithmetic coding, Shannon-Fano-Elias coding, and so on. But there is evidence that much of mathematics, perhaps all of it, may be understood as ICMUP [70]. Hence, IC techniques with a mathematical flavour may, potentially, be understood in terms of ICMUP.

In this research, seven main variants of ICMUP have been considered. They are described in [70, Section 5] and summarised in the subsections that follow.

C.2.1 Basic ICMUP

The simplest version of ICMUP is that IC may be achieved by the merging or 'unification' of two or more patterns that match each other.

C.2.2 Chunking-with-codes

This is like Basic ICMUP except that, within a given body of information, \mathbf{I} , each pattern that is produced by the unification of two or more original patterns is called a 'chunk', it is given a relatively short name or 'code', and it is placed in a separate dictionary of chunks. Then a copy of the code is put in \mathbf{I} at every place where an original pattern has been removed.

C.2.3 Schema-plus-correction

This is like chunking-with-codes except that the chunk may contain one or more places where 'corrections' may be put. A classic example is a menu in a restaurant where the menu is the chunk or schema, and choices for each of 'starter', 'main course' and 'pudding' are corrections to the schema.

C.2.4 Run-length coding

Run-length coding is where a run of two or more matching patterns is reduced to one (by unification of the several patterns) and there is something to indicate the number of patterns in the run, or simply that the pattern is repeated. An example is "Keep walking until you reach the market", where "Keep walking" indicates a sequence of steps, and "until you reach the market" indicates when to stop.

C.2.5 Class-inclusion hierarchies

A class-inclusion hierarchy is the kind of tree structure used by botanists and zoologists to classify plants and animals. It may be seen as a form of ICMUP because it saves unnecessary repetition of features. If, for example, we know that an animal is a 'mammal', we know that it has hair or fur and it feeds its young on milk. Each of those features is recorded once for 'mammal' without the need for them to be repeated for each kind of mammal.

C.2.6 Part-whole hierarchies

A part-whole hierarchy is the familiar kind of division of, for example: a car into engine, body, and wheels, and so on; the engine into block, crank shaft, cylinders, pistons, and so on.... As with class-inclusion hierarchies, a part-whole hierarchy may be seen as an example of ICMUP because it avoids unnecessary repetition of features—it is not necessary, for example, to record that the block is part of the engine, the crank shaft is part of the engine, the cylinders are part of the engine, and so on.

C.2.7 SP-multiple-alignments

The SPMA concept, described in Appendix A.3, is the seventh of the variants of ICMUP considered here. It has been shown to be a generalisation of the other six variants of ICMUP [72].

This generality is probably the main reason for the strengths of the SPTI across several broad areas (see three main links near the beginning of Appendix A).

C.3 IC in human learning, perception, and cognition

IC, and more specifically ICMUP (Appendix C.2), has been and continues to be a guiding principle in the development of the SPTI is because:

- There is substantial evidence for the importance of IC in the workings of brains and nervous systems. This strand of research was pioneered by Fred Attneave (see, for example, [2, 3]), Horace Barlow (see, for example, [5, 6]), and others, and research in this area has continued up to the present.
- There is evidence for the importance of IC in the learning of a first language [56].
- There is a review of evidence that much of HLPC may be understood as ICMUP in [69], where ICMUP is described in Appendix C.2.

C.4 IC and concepts of probability

It has been known for some time that there is an intimate relation between IC and concepts of probability [48, 49]. That intimate relation makes sense in terms of ICMUP because when two or more patterns are unified, the number of such patterns that are unified yields a measure of frequency, and that may translate into a measure of probability. Related ideas include:

- How absolute and relative probabilities are calculated for each SPMA is described in [60, Section 4.4] and [58, Section 3.7].
- The SPTI provides a framework for several kinds of probabilistic reasoning, as described in [58, Chapter 7].
- A description of how the SPTI may provide an alternative to Bayesian networks to model the phenomenon of 'explaining away' may be found in [58, Section 7.8].
- In mainstream statistics, it is normally assumed that high frequencies are needed to ensure statistical significance. But in a search for repeating patterns that may be unified to yield compression of information, the sizes of repeating patterns are as important as their frequency. Then frequencies as low as 2 or 3 may yield inferences that are statistically significant [70, Section 8.2.3].
- The close relation between IC and concepts of probability may suggest that, in developing any theory of AI or HLPC, it makes no difference whether we work from IC to probability or from probability to IC. But there are several reasons to start with IC, as described in [70, Section 8.2].

D A tentative 'tsunami' interpretation of 'wave-particle duality'

This appendix presents a tentative 'tsunami' alternative to the way in which the expression 'wave-particle duality' is generally understood.

To begin, Ball's book on quantum mechanics [4] contains a fourth chapter with the provocative title "Quantum objects are neither wave nor particle (but sometimes they might as well be)" in which he says:

"When we speak of electrons and photons, atoms and molecules, it seems perfectly reasonable to use [the word 'particle'], and I'll occasionally do so. Then we might have the image of a tiny little thing, a microscopic ball-bearing all hard and shiny. But probably the most widely known fact of quantum mechanics is that 'particles can be waves'. What then becomes of our compact little balls?" [4, Location 397].

"Quantum objects are not sometimes particles and sometimes waves, like a football fan changing her team allegiance according to last week's results." [4, Location 404].

"... all we can say is that what we measure sometimes looks like what we would expect to see if we were measuring discrete little ball-like entities, while in other experiments it looks like the behaviour expected of waves of the same kind as those of sound travelling in air, or that wrinkle and swell on the sea surface. So the phrase 'wave-particle duality' doesn't really refer to quantum objects at all, but to the interpretation of experiments—which is to say, to our human-scale view of things." [4, Locations 404–492].

It is that "the interpretation of experiments" where the concept of a 'tsunami' may be helpful. To begin, here is a description of a tsunami as it occurs in the sea:

"[A] **tsunami**, also called [a] **seismic sea wave** or **tidal wave**, ... [is] usually caused by a submarine earthquake, an underwater or coastal landslide, or a volcanic eruption.

"Origin and Development

"After an earthquake or other generating impulse occurs, a train of simple, progressive oscillatory waves is propagated great distances over the ocean surface in ever-widening circles, much like the waves produced by a pebble falling into a shallow pool. In deep water a tsunami can travel as fast as 800 km (500 miles) per hour. The wavelengths are enormous, about 100 to 200 km (60 to 120 miles), but the wave amplitudes (heights) are very small, only about 30 to 60 cm (1 to 2 feet). These long periods, coupled with the extremely low steepness and height of the waves, enables them to be completely obscured in deep water by normal wind waves and swell. A ship on the high seas experiences the passage of a tsunami as an insignificant rise and fall of only half a metre (1.6 feet), lasting from five minutes to an hour or more.

"As the waves approach the coast of a continent, however, friction with the rising sea bottom reduces the velocity of the waves. As the velocity lessens, the wavelengths become shortened and the wave amplitudes (heights) increase. Coastal waters may rise as high as 30 metres (about 100 feet) above normal sea level in 10 to 15 minutes. The continental shelf waters begin to oscillate after the rise in sea level. Between three and five major oscillations generate most of the damage, frequently appearing as powerful "run-ups" of rushing water that uproot trees, pull buildings off their foundations, carry boats far inshore, and wash away entire beaches, peninsulas, and other low-lying coastal formations. Frequently the succeeding outflow of water is just as destructive as the run-up or even more so. In any case, oscillations may continue for several days until the ocean surface reaches equilibrium." ([36, Retrieved 2020-04-15], emphasis in the original).

D.1 How the 'tsunami' interpretation would work

In brief, here is the way in which the concept of a 'tsunami' may help in the interpretation of the expression 'wave-particle duality'.

In the double-slit experiment when only one slit is open:

- Everything that is normally regarded as a subatomic particle, or even an atom or molecule, may be seen to start out like a subatomic tsunami when it is far out at sea.
- While the 'particle' is like a subatomic tsunami when it is far out at sea, it does not create much disturbance, in much the same way that "A ship on the high seas experiences the passage of a tsunami as an insignificant rise and fall of only half a metre (1.6 feet), lasting from five minutes to an hour or more."
- When the subatomic tsunami encounters a detecting device or final screen, this is somewhat like a marine tsunami when it approaches a coast: "... the wavelengths become shortened and the wave amplitudes (heights) increase. Coastal waters may rise as high as 30 metres (about 100 feet) above normal sea level in 10 to 15 minutes. ..." and great damage can be done.
- In this kind of way, the subatomic tsunami may be detected. If the detection device is a photographic plate, the interaction between the subatomic tsunami and the detection device releases energy which causes chemical changes in the photographic plate which, after development of the plate, will cause grains of silver to be deposited in a small patch. That patch of silver grains will be seen as a black spot which will itself be interpreted as the impact of a 'particle'.

• In short, the tsunami analogy suggests that the bullet-like view of subatomic particles, at least in arrangements like the double-slit experiment, is entirely an artifact of how such 'out-at-sea tsunamis' are detected.

D.2 Some other quantum experiments

Although evidence from real-life tsunamis is not as comprehensive as we might like, the concept may still be useful in the interpretation of other aspects of waveparticle duality.

In connection with the double-slit experiment, an issue which has been the subject of much discussion and research is whether there is a wave, or a bullet-like particle, travelling through the apparatus, or whether that is not determined until a measurement is taken.

D.2.1 The release of particles one at a time

When photons or other particles are released one a time, which might favour a bullet-like view of photons, interference patterns are still observed [4, Locations 768–776]. Despite the fact that it is not feasible to do relevant experiments with real tsunamis, we may say that this result with quantum phenomena is at least consistent with the 'tsunami' view of what is going on.

D.2.2 Delayed choice experiments

'Delayed choice' variants of double-slit experiments have been conceived and performed with the aim of throwing light on forms of quantum entities—'wave' or 'bullet-like particle'—as they pass through the stages of the double-slit experiment. In many variants, the results seem always to favour the view described by John Archibald Wheeler here:

"The double-slit experiment ... imposes a choice between complementary modes of observation. In each experiment we have found a way to delay that choice of type of phenomenon to be looked for up to the very final stage of development of the phenomenon, *whichever* type we then fix upon. That delay makes no difference in the experimental predictions. On this score everything we find was foreshadowed in that solitary and pregnant sentence of Bohr [7, p. 370], ' ... it ... can make no difference, as regards observable effects obtainable by a definite experimental arrangement, whether our plans for constructing or handling the instruments are fixed beforehand or whether we prefer to postpone the completion of our planning until a later moment when the particle is already on its way from one instrument to another.'" ([53, pp. 39–40], emphasis in original).

or more briefly: "No elementary quantum phenomenon is a phenomenon until it is a registered phenomenon." [54, p. 399].

D.3 'Quantum states determined by the process of detection' compared with 'the tsunami interpretation of quantum phenomena'

The idea that a given state does not exist until we apply a detection or measuring device is somewhat stronger than the 'tsunami' interpretation:

- In the first case we are saying that the phenomenon does not exist until measured or detected.
- In the latter case we are saying that, before the final stage, the phenomenon is like a tsunami that is far out to sea and that, later, anything that looks like a particle is an artifact of the detection device.

The fact that we get interference patterns when particles are released one at a time to pass through the double-slit apparatus, seems to be strong evidence that they are waves, not bullet-like objects. That suggests that the Wheeler-Bohr interpretation is too strong and that the tsunami analogy is a better fit with what is observed, in keeping with the suggestion in Appendix D that the 'tsunami' view of the double-slit experiments and related research may help to remove many of the puzzling features of quantum mechanics.

D.4 Other issues relating to the tsunami interpretation of quantum phenomena

This section considers other issues that relate to the tsunami interpretation described above.

D.4.1 Einstein's view

"It must have been around 1950. I was accompanying Einstein on a walk from The Institute for Advanced Study to his home, when he suddenly stopped, turned to me [Abraham Pais], and asked me if I really believed that the moon exists only if I look at it." [38, p. 5].
What Einstein said to Abraham Pais is very much in keeping with the puzzlement that most people experience when hearing that, in the 'Copenhagen' view of quantum mechanics, a quantum entity does not exist until it is observed.

None of this is relevant to the moon's existence because the way we perceive the moon, or anything that is bigger than quantum size, is fundamentally different from the way a subatomic particle may be detected.

Of course it makes no sense to suggest that the moon exists only if Einstein or anyone else is looking at it. At the same time, it makes a certain amount of sense to say, apart from the conclusions of Section D.3, that a subatomic particle comes into existence when it meets a detection device.

But it makes even more sense to say that: at all stages throughout a doubleslit apparatus except the last, a subatomic particle exists like a tsunami when it is far out to sea, and, finally, it creates a human-detectable artifact that looks like a particle when it encounters a detection device.

D.4.2 The detection of particles in motion

In keeping with the 'tsunami' view described above, we may suppose that, in a wire chamber, spark chamber, bubble chamber, cloud chamber, or equivalent device, any subatomic wave like an out-at-sea tsunami would initially be travelling at high speed and that, as it enters the detection device, it would gradually slow down, at the same time giving up its energy to form ions or other entities that may be interpreted as the track of a particle.

In terms of the 'tsunami' model of wave-particle duality, this would be somewhat like a tsunami wave coming to a gently-sloping shore, travelling far inland, and destroying weaker trees, less robust buildings, leaving debris, and other things that may mark its path as it goes.

D.4.3 Particles in association with other particles

What about particles such as atoms in more-or-less stable associations with other particles, in such things as molecules, metals in solid form, and crystals?

Here, it appears that 'particles' cannot be artifacts created by hitting a detector or screen in a double-slit apparatus, and they cannot be artifacts created as a 'tsunami' wave travels through a cloud chamber or equivalent device.

If the 'particle-as-artifact' analogy is to be consistent we probably need to continue with the assumption that particles in molecules, solid metals etc are basically waves but there is something about their close association which makes them behave like particles. This is weak but it does at least suggest a basis for new thinking and new research. A tentative answer here is that particles within molecules, solid metals, etc are waves on a circuit, somewhat like electrons on a circuit round the nucleus of an atom, or perhaps the circulation of electrons defines the wavelike character of atoms.

E Abbreviations

The abbreviations used in this paper are shown here in alphabetical order:

- Artificial intelligence: 'AI'.
- Commonsense reasoning and commonsense knowledge: 'CSRK'.
- Human learning, perception, and cognition: 'HLPC'.
- Information compression: 'IC'.
- Information compression via the matching and unification of patterns: 'ICMUP'.
- Mathematical Universe Hypothesis: 'MUH'.
- Non-Quantum Parallel Processing: 'NQPP'.
- Quantum mechanics: 'QM'.
- SP Computer Model: 'SPCM'.
- SP-multiple-alignment: 'SPMA'.
- SP Theory of Intelligence: 'SPTI'.

As noted in Appendix C.1, it is intended that 'SP' should be treated as a name, not an abbreviation for *Simplicity* and *Power*.

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